Interbank Networks Formed through Money Creation, Too Connected to Fail and Systemic Risk

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Abstract

This paper presents a new approach to endogenize interbank credit networks, based on banks' specialty that their liabilities are accepted as a means of payment. This approach takes into account how borrowing on banks' asset side affects depositing on their liability side in general equilibrium. This approach is applied to endogenize a star structured interbank network with the aim of studying the issues of too connected to fail (TCTF) and systemic risk. In the model the banking system melts down on equilibrium path. Moreover, (1) the resources are inefficiently concentrated at the center of the network and the higher the interbank rate, the worse the efficiency; (2) the network of interbank credit alone has no issue of TCTF, which, however, arises if interbank insurance is also introduced; (3) this insurance becomes cheaper if the interbank rate is higher and it makes loss if the rate goes above a threshold; (4) early news matters more for systemic stability than late one; and (5) there is a zone of news based on which the event of system meltdown is likely to happen, but has not happened yet. This type of news provides an early warning of the event.

Key words: too connected to fail, systemic risk, interbank credit networks, interbank insurance, early warning zone, system meltdown

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1 Introduction

The 2008-9 financial crisis highlights the importance of interconnections between banks for the systemic stability. One of the issues is "Too Connected To Fail" (TCTF), that is, if one bank that is connected to many other banks fails, its failure might cause such a severe loss to them as to bring them all down to insolvency, making a systemic event.[1] This paper studies this issue with a new approach to endogenize interbank credit networks. This approach is based on a specialty of banks, namely that their liabilities, especially those in the form of demand deposit, are widely accepted as a means of payment. Thereby banks finance lending through money creation, that is, by issuing liabilities. Due to this specialty, an interbank credit link can be formed *passively*. Suppose a firm borrows from bank A and uses the borrowed *money* – which is the bank's liability – to purchase resources from a seller that deposits with bank B. Then when *this money* as the sales proceeds is deposited into bank B, the bank holds this liability of bank A and thus owns a credit position to it.[2] This way of forming interbank credit links has not been considered in the literature on interbank networks, which models banks as Intermediaries of Loanable Funds, ILF hereafter.[3] Compared to the ILF based approaches which the literature takes, the approach of this paper has two merits. First, it captures the fact that a substantial part of funds deposited into the banking system comes from what is borrowed out of it. This interaction between borrowing and depositing might not matter much when single banks are concerned, but it is important when we consider the banking system as a whole. Second, the approach of this paper accommodates multiple goods, whereas the ILF approaches typically deal with only one good, usually called funds. In this paper, the means of payment that is borrowed from the banking system is used to buy multiple types of resources, which can represent labor,

[1] The issue of TCTF could alternatively be due to the adverse effects on the liability side, that is, if the "too connected" bank fails it might call the banks with which it is linked to immediately settle their liabilities to it, which could cause a liquidity stress on them. This alternative side of the TCTF is not addressed in this paper, which abstracts from the liquidity issues of banks.

[2] This way of producing interbank liabilities is also noticed by Piazzesi and Schneider (2017), which, however, is not concerned with banking networks.

[3] Both terms of "financing through money creation" and ILF are due to Jakab and Kumhof (2015), who provide detailed discussion on differences between these two approaches of modeling banks. See also Wang (2014) for a general equilibrium analysis of money creation by banks and its implications for central banking.

land, machinery, knowledge, etc. Indeed, this paper shows that a bank's position at the interbank credit network is determined by the type of resources that its depositors own.

This paper's approach can be used to generate a variety of interbank networks, depending on the distribution of resources and investment projects.[4] Considering it focuses on the issues of TCTF and systemic risk, it considers a model economy where the equilibrium interbank network is of the star structure, the simplest structure to consider these issues. The model economy consists of many regions, each of which is populated by a bank, a continuum of entrepreneurs and a continuum of households. Households are endowed with two types of resources. One type is to be found only at one particular region called capital, while the other type exists at all the other regions, called provinces. The former type is meant to represent resources that concentrate on economic centers, such as convenience for trading[5] or know-how[6], while the latter type represents resources that are more dispersed in the whole economy, such as labor or land. Entrepreneurs have technology to use the two types of resources to produce the consumption good, corn. To obtain resources, yet entrepreneurs face a friction of payment. They cannot use their own promise to pay, but have to borrow banks', to buy resources from households. This assumption captures the aforementioned speciality of banks, namely that their IOUs are widely accepted as a means of payment, while those of non-bank firms or individual persons are not. Therefore, in this economy, entrepreneurs and households have real resources, but these resources can be put together to produce the consumption good only with what is supplied solely by banks, namely, means of payment.

In this economy, entrepreneurs move around to buy resources using their regional banks' promise to pay, which the sellers of the resources then deposit into their local banks, thus formed interbank liability links. Assume that all the provinces are symmetric. As a result, the liabilities between them are canceled out and all the interbank liabilities are between them and the capital bank. Hence, the interbank credit network is of a star structure. The key question

[4] For example, if firms that borrow from bank 1 need to buy a intermediate good from bank 2's depositors, who, for the production of the good, buy an input good from bank 3's depositors, then there will be a chain of interbank liabilities in which bank 1 owes to bank 2 and bank 2 to bank 3.

[5] Observe that economic centers typically develop from a privileged geographical position as a nexus of transportation networks, such as Chicago, New York, St Louis of the U.S. and Wuhan, Shanghai of China.

[6] One reason for people with high human capital to concentrate in one place is the externalities between them, as is model by Lucas (1988).

is: who owes to whom? If the capital bank owes to all the provincial banks, then the capital bank could be TCTF because its failure could reduce the asset value of all the provincial banks and thereby bring them down. If, the other way around, the provincial banks owe to the capital bank, then the interbank network has no issue of TCTF because the failure of the capital bank would have no impact on the asset value of the provincial banks.

The answer to that key question, if an ILF approach was applied, would depend on the assumption on the relative abundance of funds available to banks, given the setting of their asset side – namely entrepreneurs' technology: If the capital bank is in deficit of funds and the provincial banks in surplus, the former borrows from the latter, and vice versa. That is because the ILF approaches, which the literature on financial networks takes, do not accommodate the aforementioned general equilibrium effect of borrowing for depositing. By contrast, this effect is captured in the present paper, where funds borrowed by entrepreneurs are portioned out, through market mechanics, into the sales proceeds of the resources at each region, and thereby determine the quantity of the funds deposited into each bank. Given the entrepreneurs' technology, which determines their scale of borrowing, therefore, there is only one way in which the interbank liabilities go. In particular, it is always provincial banks owing to the capital bank, not the other way around, if their number is large enough, namely, if the capital bank is sufficiently connected. Interbank liabilities alone, hence, do not drive the issue of TCTF, showing which is the first contribution of this paper.

Its second contribution is to show that the capital bank offers cheaper loans than provincial banks, whereby the resources are inefficiently concentrated at the capital, and moreover, the higher is the interbank interest rate, the more resources are concentrated at the capital. The intuition is as follows. A fraction of promise to pay that a bank loans out to the entrepreneurs flows into other banks and becomes a liability to them which incurs costs of interbank interest to the issuing bank. Relative to the capital bank, a provincial bank sees a greater fraction of its issues to become interbank liabilities, and therefore bears a greater marginal cost of lending. This leads a provincial bank to charge a higher interest rate on its loans than the capital bank does. And the higher the interbank rate, the larger the difference in marginal cost between the two banks, and the larger the difference in the rate charged on loans, causing more resources concentrated at the capital.

As the network of interbank credit alone has no issue of TCTF, to consider the issue, we introduce another type of interbank claims: interbank insurance. The model is thus extended to include independent shocks to banks' loans. Namely, with a certain probability loans to entrepreneurs will not perform leaving the banks insufficient revenue to redeem liabilities. To avoid this costly insolvency, banks demand insurance against the idiosyncratic risks.

The third contribution of this paper is to consider the interaction between these two types of interbank claims: credit and insurance. This interaction generates two effects. First, the position of the capital bank in the interbank *credit* network gives it a natural advantage to be the provider of the interbank *insurance* This insurance is done, ultimately, by pooling as many independent risks as possible. A provincial bank being the insurance provider suffers the problem of miscoordination. That is, if a bank decides to buy insurance from the provincial bank, the insuree bank might end up with being the sole buyer of it, thereby obtaining not much insurance, as only two risks are pooled. In contrast, this mis-coordination problem evaporates if the capital bank provides the insurance because the bank receives interbank credit repayments from all the provincial banks and on its asset side the risks have been pooled. Therefore, even if a bank is the sole buyer of insurance from the capital bank, the insuree bank will obtain the full insurance repayment unless in the extremely rare event when a large fraction of provincial banks default on their interbank credit payments to the capital bank.

The second effect of the interaction between the two types of interbank claims is that the higher the interbank interest rate, the lower the insurance premium that the capital bank charges. This negative relationship arises because, to make lending, provincial banks need to buy insurance to cover for the negative shock. If the interbank rate is higher, the capital bank wants to lower the insurance premium in order to encourage provincial banks to increase lending and their liabilities to the capital bank. If the interbank rate is above a threshold, actually, the insurance is sold at a negative profit margin to the capital bank. In this case buying insurance itself gives value to provincial banks, which then buy as much as possible. Therefore, the quantity of insurance that peripheral banks buy jumps discontinuously at a certain value of the interbank rate.

The capital bank, being the sole provider of insurance to all the provincial banks, is now TCTF: Its failure means insufficient insurance to all the provincial banks, which might consequently face too high a default risk. To explore this intuition, the model is extended further in the last part of this paper. Suppose that provincial banks' shocks are revealed sequentially. At any point of time, conditional on the shocks revealed thus far, depositors (namely households) assess the probability in which the capital bank will so severely default on its insurance obligations that provincial banks default under the negative shocks. If this probability is greater than a threshold, run occurs to all banks except those which are known to have received positive shocks. This can be regarded as an event of system meltdown. This event occurs on equilibrium path, namely if a large fraction of revealed shocks are negative. Moreover, given the same final outcome, whether the event occurs may depend on the order in which the shocks are revealed. It occurs if negative shocks are front-loaded, but not if positive shock are. Therefore, early information matters more for the systemic stability than later one. Lastly, this paper finds that there exists a set of revelations based on which the event of system meltdown is very likely to happen, but has not happened yet. These revelations give an early warning of the system meltdown. Showing the existence of this warning zone and importance of earlier revelations is the fourth contribution of this paper.

This paper contributes to the literature that considers the implications of networks of financial claims for systemic stability, for a survey of which see Allen and Babus (2009), Bougheas and Kirman (2014), Cabrales et al (2015), and Glasserman and Young (2015). Most of the studies in this literature takes the financial networks as exogenously given. More closely related to the present paper are those studies in which the financial networks arise endogenously; see Acemoglu et al (2014), Allen et al (2012), Babus (2016), Farboodi (2015) and Zawadowski (2013). Allen et al (2012) derive two structures of interbank networks in equilibrium and show that the systemic risk they generate critically depends on the banks' funding maturity. Both Acemoglu et al (2014) and Farboodi (2015) consider the trade-off between the benefit of investment opportunities and the cost of possible contagion that an extra interbank link brings about.[7] Both Acemoglu et al (2014) and Zawadowski (2013) demonstrate that inefficiency arises due to financial network

[7] This trade-off, in a reduced form, is also studied by Blume et al (2013) and Erol and Vehra (2014). The latter furthermore shows that the probability of system-wide default increases with the probability of good shocks, counterintuitively, which underlines the importance of taking into account the endogeneity of forming interbank links. Moreover, Glasserman and Young (2015) survey the studies on a similar trade-off, between the benefit of diversification and the cost of possible contagion.

externalities, namely that a bank fails to internalize the implications of its decision for the banks with whom it is not directly linked, the decision concerned with forming interbank links in the former study and with buying insurance against counterparty risks in the latter. Babus (2016), based on Allen and Gale (2000), endogenizes a networks of banks providing mutual insurance against the liquidity risks, and show that the equilibrium networks bear a small or even nil systemic risk. In terms of commonality, Allen et. al. (2012), Babus (2016), Zawadowski (2013) and the present paper all underline the importance of interbank insurance. And both Farboodi (2015) and the present paper endogenize a core-peripheral network, while the structure derived in the former is richer.

Relative to this literature, the fundamental innovation of this paper is that its approach to interbank credit networks are based on banks' function of money creation, whereas the literature takes approach that model banks as intermediaries of loanable funds, a real good. In addition, this paper considers two types of interbank claims – credit and insurance – and their interplay, whereas only one of these two is considered in the literature. With these two innovations, this paper makes the four contributions, stated above, to the literature.

In this paper, the interbank credit network is formed by entrepreneurs moving around. In the same spirit, it is formed by depositors moving around in Freixas et. al. (2000). The two papers, however, have different focuses. The present paper is concerned with the issue of too connected to fail. In contrast, their paper is concerned with the vulnerability of the interbank networks to mis-coordinated withdraw by depositor in the spirit of Diamond and Dybvig (1983): the over-withdraw from one bank weakens the bank and thereby disables it from providing credit to other banks, which might trigger a mass withdraw to them and thereby get them weakened, eventually causing system meltdown.

In the present paper, the sufficient accumulation of negative individual shocks triggers run to all the banks whose states have not been revealed. This feature is also present in Chen (1999) and the present paper is therefore related to other studies that examine information contagion with the networks of financial claims set in the background; see Acharya (2009), Acharya and Yorulmazer (2007, 2008), Ahnert and Georg (2017), Dasgupta (2004) and Leitner (2005), among others, and see Benoit et. al. (2017) for a survey. Relative to this literature, the present paper specifically considers the issue of too connected to fail. Furthermore, in the present paper, the contagion occurs neither because of exogenous common exposure (such as in Chen 1999), nor because of the counterparty risk (such as in Ahnerta and Georg 2017) as the negative-shocked banks are not directly linked to the banks with unknown shocks. Yet, it has a flavor of both. On the one hand, all provincial banks buy insurance from the capital bank, which thus can be regarded as a common factor for provincial banks. On the other hand, this common factor is not an exogenous element, but a county-party in the interbank network.

The rest of the paper is organized as follows. Section 2 sets up the baseline model to demonstrate the mechanics of the paper's approach. It is solved in Section 3 and extended in Section 4 with the interbank insurance introduced. It is further extended and modified in Section 5 to endogenize the event of system meltdown. Finally, Section 6 concludes. Proofs of technical importance are relegated in Appendix.

2 The Baseline Model

The economy lasts for two periods, $t = 0$ for production and $t = 1$ for consumption. It is composed of one capital city and N provinces, with $N \geq 2$. Each of these regions is populated by one bank, a continuum of $[0, 1]$ entrepreneurs, and a continuum of households. All the economic agents are risk neutral and protected by limited liability. Each entrepreneur has h units of human capital. The households at a province own X_P units of type 1 resources. The households at the capital city own X_C units of type 2 resources. Entrepreneurs use the two types of resources and their human capital to produce corn, the consumption good, also used as the numeraire. The production function a region $i = C, P$, which denotes the capital and a province respectively, is:

$$y = A_i h^{1-\alpha} \left(x_1^{\beta} x_2^{1-\beta} \right)^{\alpha}.$$

Normalize $h = 1$. Note that while type 1 resources are dispersed in the economy, type 2 resources are to be found nowhere but one particular region, namely the capital. This type of resources therefore represents those resources that concentrate on a limited number of places, such as convenience for trade and shipment or political connection, which abounds at the political centers.

Households are willing to give up their resources at $t = 0$ only in the hope of being repaid with corn at $t = 1$. That is, as in a typical circumstance concerned with finance, at $t = 0$, they exchange resources for a promise that they will be paid back with corn at $t = 1$. This exchange is feasible if and only if they trust the promise, namely, they accept this promise as a means of paying for their resources. If households accept entrepreneurs' promise to pay, banks would play no role in the economy. What makes banks matter is the following friction.

Assumption K1: households do not accept entrepreneurs' promises to pay, but accept banks', as a means of paying for their resources.

This assumption captures the aforementioned specialty of banks, that their promise to pay – namely their liability – is *widely* accepted as a means of payment, whereas rarely so is the promise to pay of non-bank firms or that of individual persons.

Due to this assumption, to buy resources from households, entrepreneurs have to borrow some banks' promise to pay. To fix the idea, let this promise be recorded on a *note*. Hence, a note issued by a bank X reads: "Bank X promises to pay the bearer of this note quantity Y of corn at $t = 1$". This quantity is the face value of the note. Banks charge interest on lending. If at $t = 0$ an entrepreneur borrows a bank's notes of face value F at interest rate r and uses them to buy resources from some households, then the entrepreneur owes $F(1 + r)$ units of corn to the bank and the bank owes F units of corn to the households, and at $t = 1$, these debts are settled by him paying $F(1 + r)$ units of corn to the bank and the bank paying F units of corn to the households. Hence, if a bank's notes are used to buy some resource, the price of the resources is the face value of the notes in exchange for one unit of the resources, namely, the quantity of corn that the bank will pay for the unit.

In each region there exist economic agents who do not possess any genuine entrepreneurial human capital but want to do projects for their own private benefit. As a result, banks need to screen borrowers before lending to them. We assume that each bank is specialized to screening the entrepreneurs of its own region. As such, a region in the model can also represent, besides a geographic area, a sector, industry, or business field, whereby the region's bank represents the bank that is specialized to it. This specialization of banks has been well documented in empirical research, e.g. by Jonghe et. al. (2016), Liu and Pogach (2016), Ongena and Yu (2017), and Paravisini et. al. (2014). Specifically, we assume that it costs a bank c_l to screen an entrepreneur within its own region and c_h to screen one without. To simplify the exposition, furthermore, we assume $c_l = 0$ and c_h is high enough – the exact meaning of which will be given later – that in attracting entrepreneurs of a region the local bank out-competes an outside bank even if the former charges the monopolistic price for its funding. As a result, entrepreneurs borrow only from their local banks.

Entrepreneurs, after borrowing notes from their local banks, move around to buy resources. They go to regions with the lowest price of resources with an equal probability. After the trading, bank notes end up in the hand of households. We assume that due to concerns of safety, households deposit all the notes in their holding with their local banks. This process generates interbank credit. For example, suppose that some households deposit into the capital bank the notes of a provincial bank with face value F. Then, on the liability side of the capital bank, newly added is a liability to these households and on the asset side, newly added is a credit position to the provincial bank. If the interbank interest rate is $\rho \geq 0$,[8] the depositing changes the capital bank's balance sheet into the following.

Assets	Liabilities
Old assets (X)	Old liabilities (X)
Credit to the provincial bank ($F \times (1+\rho)$)	Liability to the depositors of these notes (F)
	Gain to the equity $F \times \rho$

Table 1: The balance sheet of the capital bank with the provincial bank's notes deposited in

The timing at $t = 0$ is as follows.

1. Banks at a region $i = C, P$ post the interest rate they charge, r_i.

2. Entrepreneurs at a region $i = C, P$ borrow face value m_i of notes from their local banks.

3. Entrepreneurs move around to buy the two types of resources with the notes borrowed. Let p_k be the price of type k resources for $k = 1, 2$.

4. Households deposit with their local banks the notes for which they have exchanged their resources.

5. Banks net out the liabilities between them.

[8] This interbank interest rate is certainly endogenous, which, however, is beyond the scope of this paper. That is because thus far we have not introduced the liquid assets that are used to settle the interbank liabilities, namely bank reserves. If these liquid assets are introduced, then the return rate that they earn will equal the interbank interest rate in equilibrium.

The timing at $t = 1$ is as follows.

1. Entrepreneurs produce corn and settle their debts to the local banks by repaying $m_i(1+r_i)$ units of corn.

2. Banks use corn paid in by the entrepreneurs to settle the net interbank liabilities and redeem notes from households.

3. Agents consume corn that they have obtained.

We consider only the symmetric equilibrium in which all provincial banks make the same decision, defined as follows.

Definition 1 *A profile* $(m_P, m_C, r_P, r_C, p_1, p_2)$ *forms an equilibrium, if for* $i = C, P$, *(i) given* (r_i, p_1, p_2) , m_i *is the optimal demand of notes by entrepreneurs at region* i; *(ii) given* (p_1, p_2) *and the entrepreneurs' demand function* $m_i(r_i; p_1, p_2)$, r_i *is the optimal rate charged by the bank at region* i; *and (iii) the markets for both types of resources clear.*

In the symmetric equilibria, the liabilities between the provincial banks cancel each other. Therefore, all the net interbank positions are between a provincial bank and the capital bank. Put differently, the interbank liability network is of a star structure. The key question is: who owes to whom? Two scenarios could arise. One, the capital bank owes to all the provincial banks. The other, provincial banks all owe to the capital bank. In the first scenario, the capital bank might be Too Connected To Fail (TCTF) because if due to some exogenous reason it fails, then its failure might reduce the asset value of all the provincial banks so much as to bring them all into failure. In the second scenario, however, the network has no issue of TCTF, because the failure of the capital bank would incur no loss to any provincial banks.[9] Which of these two scenarios arises in equilibrium is analyzed in the next section.

[9] In this scenario, however, the failure of the capital bank could still cause liquidity issues to provincial banks by demanding immediate settlement of their liabilities to it. This aspect of TCTF is abstracted from in the paper as the issue of bank liquidity in general is.

3 Who Owes to Whom

To analyze the model, we first consider entrepreneurs' decision, which is on the quantity of money to borrow. When making this decision, they take as given the interest rate that their local banks charge, r_i, and the prices of the two types of resources, (p_1, p_2). Hence entrepreneurs at region $i = C, P$ solve

$$\max_m A_i \left(x_1^\beta x_2^{1-\beta} \right)^\alpha - m(1 + r_i), \tag{1}$$
$$s.t. \quad m = p_1 x_1 + p_2 x_2,$$

At the optimum

$$m = m_i(r_i; p_1, p_2) := \left(\frac{A_i \alpha}{1 + r_i} \right)^{\frac{1}{1-\alpha}} \left(\beta^\beta (1 - \beta)^{1-\beta} \right)^{\frac{\alpha}{1-\alpha}} \left(p_1^\beta p_2^{1-\beta} \right)^{-\frac{\alpha}{1-\alpha}}, \tag{2}$$

and as is well known with a Cobb-Douglas function, β fraction of the budget is spent on type 1 resources and $1 - \beta$ fraction on type 2. Thus, the quantity of type $k = 1, 2$ resources demanded by entrepreneurs of region $i = C, P$, denoted by x_k^i, is given by:

$$x_k^i = \frac{\beta m_i(r_i; p_1, p_2)}{p_k}.$$

Now consider the decision of banks. Based on Table 1, at the end of $t = 0$, a bank's balance sheet is as follows.

Assets	Liabilities
Loans to entrepreneurs: π	to bearers of its own notes deposited: F_{own}
Interbank credit position: $F_{other}(1 + \rho)$	to bearers of other banks' notes deposited: F_{other}
	Other banks that hold its notes: $\widetilde{F}_{own}(1 + \rho)$
	Equity: E

Table 2: a bank's balance sheet at the end of $t = 0$

Here F_{other} is the aggregate face value of other banks' note deposited with this bank and $\widetilde{F}_{own}$ is that of its own notes deposited with other banks. Thus $F_{other} - \widetilde{F}_{own}$ is the net interbank credit position of the bank, denoted by Υ. If this bank lends out face value M of notes in total at interest r, then $\pi = M(1 + r)$. As these notes are either deposited back into the bank or flow to other banks, $F_{own} + \widetilde{F}_{own} = M$. Therefore, the value of the bank $E = \pi + F_{other}(1+\rho) - F_{own} - F_{other} - \widetilde{F}_{own}(1+\rho) = M(1+r) + \left(F_{other} - \widetilde{F}_{own}\right)\rho - \left(F_{own} + \widetilde{F}_{own}\right)$. With $F_{own} + \widetilde{F}_{own} = M$ and $F_{other} - \widetilde{F}_{own} = \Upsilon$, the bank's value is

$$E = Mr + \Upsilon\rho. \tag{3}$$

Intuitively, this equation says a bank's profit comes either from the interest of its loans to the entrepreneurs or from its interbank credit position.

Consider now a representative provincial bank's decision on the scale of issuance M, given that the capital bank issues M_C, and all the other provincial banks issue $\widetilde{M}$. If it charges interest r on loans, then $M = m_P(r; p_1, p_2)$ given by (2). Its net interbank credit position Υ_P equals what is deposited in, denoted by D_P, minus what it issues, M. The former comes from the sales revenue of the local households. To calculate this revenue, observe that entrepreneurs all spend β fraction of their borrowing on type 1 resources, $1/N$ of which flows to the representative provincial bank. Therefore, the quantity of funds deposited into the bank D_P equals β/N of the aggregate borrowing, $(N-1)\widetilde{M} + M + M_C$, that is,

$$D_P := \frac{\beta\left((N-1)\widetilde{M} + M + M_C\right)}{N}. \tag{4}$$

As $\Upsilon_P = D_P - M$, it follows that

$$\Upsilon_P = -\frac{N-\beta}{N}M + \frac{\beta\left((N-1)\widetilde{M} + M_C\right)}{N}. \tag{5}$$

The above analysis demonstrates how funds borrowed by entrepreneurs on banks' asset side, $(N-1)\widetilde{M} + M + M_C$ in aggregation, are portioned out through market mechanics into the sales proceeds of the households of a particular region, e.g. $\left((N-1)\widetilde{M} + M + M_C\right) \times \beta/N$ for a province, which are then deposited into the region's bank. In the model economy, therefore, the quantity of funds deposited into each of the banks on their liability side is determined by the borrowing decisions of entrepreneurs on the asset side. This determination captures, in a stylized way, the observation that in real life a large part of the funds deposited into the banking system comes from what is borrowed out of it.

The bank's profit is $Mr + \Upsilon_P\rho$ as given by (3). With Υ_P found in (5) and let $M' := (N-1)\widetilde{M} + M_C$ denote the aggregate lending by all the other banks, the representative provincial bank's problem is:

$$\max_r M \times (r - \frac{N-\beta}{N}\rho) + \frac{\beta M'}{N}\rho,$$
$$s.t. \quad M = m_P(r; p_1, p_2).$$

As M' is beyond the bank's control, this problem is thus equivalent to:

$$\max_r m_P(r; p_1, p_2) \times (r - \frac{N-\beta}{N}\rho).$$

When the entrepreneurs decide their demand for the bank's note, they take prices p_1 and p_2 as given, that is, they do not take into account the effect of their demand on the prices. Therefore, in the above problem, (p_1, p_2) is taken as given. With m_P given by (2), therefore, the provincial bank's problem is equivalent to

$$\max_r \left(\frac{A_P}{1+r}\right)^{\frac{1}{1-\alpha}} \times (r - \frac{N-\beta}{N}\rho).$$

This problem solved, the optimal interest rate that a provincial bank charges is thus:

$$r_P = \frac{1-\alpha+\frac{N-\beta}{N}\rho}{\alpha}.$$

Substitute it back to (2) and the size of the bank's lending is

$$M_P = \left(\frac{A_P\alpha}{1+\frac{N-\beta}{N}\rho}\right)^{\frac{1}{1-\alpha}} \left(\beta^\beta (1-\beta)^{1-\beta}\right)^{\frac{\alpha}{1-\alpha}} \left(p_1^\beta p_2^{1-\beta}\right)^{-\frac{\alpha}{1-\alpha}} := \widetilde{M}_P(p_1, p_2). \tag{6}$$

In a similar way we consider the capital bank's problem. If it sets the interest rate to be r, then it issues $M_C = m_C(r; p_1, p_2)$, also given by (2). What the local households deposit into it is the value of type 2 resources, on which $1-\beta$ fraction of the aggregate lending is spent. Thus

$$D_C = (1-\beta)(M' + M_C), \tag{7}$$

where M' denotes the total lending of all the other banks. As $\Upsilon_C = D_C - M_C$, we have

$$\Upsilon_C = -\beta M_C + (1-\beta) M'. \tag{8}$$

With the bank's value being $Mr + \Upsilon\rho$, the capital bank's problem is then:

$$\max_r M_C(r; p_1, p_2) \times (r - \beta\rho) + (1-\beta) M'\rho,$$

In a similar way, this problem is found to be equivalent to:

$$\max_r \left(\frac{A_C}{1+r}\right)^{\frac{1}{1-\alpha}} \times (r - \beta\rho).$$

The optimal interest rate that the capital bank charges is thus

$$r_C = \frac{1-\alpha+\beta\rho}{\alpha},$$

which induces the aggregate demand for its notes to be:

$$M_C = \left(\frac{A_C\alpha}{1+\beta\rho}\right)^{\frac{1}{1-\alpha}} \left(\beta^\beta (1-\beta)^{1-\beta}\right)^{\frac{\alpha}{1-\alpha}} \left(p_1^\beta p_2^{1-\beta}\right)^{-\frac{\alpha}{1-\alpha}} := \widetilde{M}_C(p_1, p_2). \tag{9}$$

To fully determine the equilibrium, we are left to find out the equilibrium prices of the two types of resources. As was said, β fraction of the aggregate lending is spent on type 1 resources, and $1-\beta$ on type 2. Therefore,

$$\begin{aligned} \beta\left(N\widetilde{M}_P(p_1,p_2) + \widetilde{M}_C(p_1,p_2)\right) &= p_1 \times NX_P \\ (1-\beta)\left(N\widetilde{M}_P(p_1,p_2) + \widetilde{M}_C(p_1,p_2)\right) &= p_2 \times X_C.^{10} \end{aligned}$$

To find out who owes to whom, we shall calculate the sign of the capital bank's net interbank position, Υ_C, or equivalently, that of a provincial bank's position, Υ_P, as these two types of positions cancel each other, that is, $\Upsilon_C + N\Upsilon_P = 0$.[11] By (8)

$$\Upsilon_C = -\beta M_C + (1-\beta) N M_P,$$

which can be intuitively understood as follows. A fraction $1-\beta$ of the aggregate issues of by the provincial banks, NM_P, flows to the capital bank after the provincial entrepreneurs buy type 2 resources at the capital city, while a fraction β of the issues by the capital bank, M_C, flows to the provincial banks after the capital entrepreneurs buy type 1 resources at the provinces. The net credit position of the capital bank is the former subtracting the latter. Therefore, $\Upsilon_C > 0$, namely, all the provincial banks owe to the capital bank, if and only if $(1-\beta) NM_P > \beta M_C$, which, with (6) and (9), is equivalent to $(1-\beta) N \left(\frac{A_P\alpha}{1+\frac{N-\beta}{N}\rho}\right)^{\frac{1}{1-\alpha}} > \beta \left(\frac{A_C\alpha}{1+\beta\rho}\right)^{\frac{1}{1-\alpha}}$. With some re-arrangement, we find that $\Upsilon_C > 0$ if and only if

$$\frac{A_P}{A_C} > \left(\frac{\beta}{1-\beta}\right)^{1-\alpha} \frac{1+\frac{N-\beta}{N}\rho}{(1+\beta\rho) N^{1-\alpha}}. \tag{10}$$

[11] By (5) $\Upsilon_P = -\frac{N-\beta}{N} d_P + \frac{\beta}{N}((N-1) d_P + d_C)$ and by (5) $\Upsilon_C = -\beta d_C + (1-\beta) N d_P$. Hence, $\Upsilon_C + N\Upsilon_P = -\beta d_C + (1-\beta) N d_P + [-(N-\beta) d_P + \beta((N-1) d_P + d_C)] = 0$.

From (10), we see that this paper's approach re-captures some intuitions that could be derived with the ILF approaches that the literature uses. With those approaches, if the capital bank has relatively scarcer investment opportunities than provincial banks, other things fixed, then the capital bank lends funds to provincial banks, that is, $\Upsilon_C > 0$. In the model economy, the number of entrepreneurs in each region is the same – a continuum of $[0, 1]$ – and rather the scale of the investment opportunities is measured by the productivity parameter because a higher productivity induces greater investment. Therefore, the above intuition suggests that if A_P/A_C is large enough, then $\Upsilon_C > 0$, which is confirmed according to inequality (10).

Moreover, a new insight is shed with inequality (10). Suppose we fix the interbank interest rate ρ and the setting on banks' asset side, which consists of entrepreneurs' technologies represented by parameters A_P, A_C and β. Then, the direction of the interbank liabilities is determined: $\Upsilon_C > 0$ if equation (10) holds and $\Upsilon_C < 0$ if the inverse inequality holds. By contrast, with the ILF approaches, if the setting on banks' asset side and interbank rate are fixed, there is still a degree of freedom in assuming the relative abundance of funds on the liability side. If the capital bank has way less funds than the provincial banks, then it borrows funds from the latter and $\Upsilon_C < 0$, while if the former has way more funds than the latter, then it lends funds to the latter and $\Upsilon_C > 0$. That is, the direction of the interbank liabilities can still go both ways. This degree of freedom, in contrast, is not there in the model economy, because here the quantity of funds deposited into each bank is determined by the setting on banks' asset side, as shown above, whereas this effect of borrowing on depositing is not captured with the ILF approaches.

To elicit another insight of this section, observe that (10) is equivalent to

$$N\left(\lambda N^{1-\alpha} - 1\right) > \left(N - \beta - \beta\lambda N^{2-\alpha}\right)\rho, \tag{11}$$

where $\lambda := \left(\frac{1-\beta}{\beta}\right)^{1-\alpha} \frac{A_P}{A_C} > 0$. Therefore, $\Upsilon_C > 0$ if and only if (11) holds true. Given (A_P, A_C, β), namely given (λ, β), $N - \beta - \beta\lambda N^{2-\alpha} < 0$ for a large enough N. Therefore, $\widehat{N} := \inf\{N' | N - \beta - \beta\lambda N^{2-\alpha} < 0 \text{ for any } N > N'\}$ is well defined. Let $N^* := \max\left(\left(\frac{1}{\lambda}\right)^{\frac{1}{1-\alpha}}, \widehat{N}\right)$, which is finite (unless $A_P = 0$ and thus $\lambda = 0$). If $N > N^*$, then $N > \left(\frac{1}{\lambda}\right)^{\frac{1}{1-\alpha}}$ and hence the left hand side of (11) is positive, and also $N > \widehat{N}$ and hence the right hand side is negative for any $\rho \geq 0$. Therefore, (11) holds true – and thus $\Upsilon_C > 0$ – for any $\rho \geq 0$. The following proposition is thus self-evident.

Proposition 1 *Given* (A_P, A_C, β) *with* $A_P > 0$, *there exists* N^* *such that if* $N > N^*$, *then* $\Upsilon_C > 0$ *for any* $\rho \geq 0$. *That is, if the capital bank is sufficiently connected, then it owes to all the provincial banks and the equilibrium interbank credit network has no issue of too connected to fail.*

By this proposition, to study the issue of too connected to fail, considering the interbank credit claims alone is not sufficient and another type of interbank claims need to be introduced. Among natural candidates for it is the insurances against idiosyncratic risks. That is not only because interbank insurance claims are important, but also because by modeling these risks we can investigate under which conditions the accumulation of idiosyncratic risks lead to systemic risk. The idiosyncratic risks and the insurance against them will be introduced in the next section.

Passing on to that, however, we deliver the last insight of this section, that resources are over-concentrated at the capital city if the capital bank is sufficiently connected. To present this insight in the simplest way, let us focus on the special case in which $A_P = A_C$, that is, all the entrepreneurs have an identical production technology. In this case, the socially optimal allocation is the one that gives the capital entrepreneurs and the provincial ones the same quantity of both types of resources, that is,

$$\frac{\widehat{x}_1^C}{\widehat{x}_1^P} = \frac{\widehat{x}_2^C}{\widehat{x}_2^P} = 1, \tag{12}$$

Consider now the equilibrium allocation. All the entrepreneurs spend borrowed funds in the same way, β fraction on type 1, $1-\beta$ on type 2. Therefore,

$$\frac{x_1^C}{x_1^P} = \frac{x_2^C}{x_2^P} = \frac{M_C}{M_P}.$$

With M_P given in (6) and M_C in (9), and $A_P = A_C$, the equilibrium allocation is:

$$\frac{x_1^C}{x_1^P} = \frac{x_2^C}{x_2^P} = \left(\frac{1 + (1-\beta/N)\,\rho}{1+\beta\rho}\right)^{\frac{1}{1-\alpha}}. \tag{13}$$

A comparison between (12) and (13) shows that if $\rho > 0$, unless in the very special case where $1-\beta/N = \beta$, that is, $N = \beta/(1-\beta)$, the equilibrium allocation differs to the socially optimal one and is inefficient. Intuitively, when a bank lends out its notes, a fraction of them flows to other banks and becomes interbank liabilities, on which the bank pays out interest at rate $\rho > 0$. The higher is this fraction, thus, the higher the marginal cost of lending and as a result the higher the interest rate charged. For a provincial bank, out of one unit of notes that it lends out, β fraction of them is used to buy type 1 resources. Out of this β unit of notes, only $1/N$ of them – that is β/N unit – flows back to the issuer bank. Therefore, if a provincial bank lends out one unit of notes, fraction $1 - \beta/N$ of them becomes interbank liabilities, whereby its marginal cost of lending is $(1 - \beta/N)\,\rho$. Similarly, if the capital bank lends out one unit of notes, then fraction β of them flows to the provinces being used to buy type 1 resources, whereby its marginal cost of lending is $\beta\rho$. This explains $r_P = [1 - \alpha + (1 - \beta/N)\,\rho]\,/\alpha$ and $r_C = [1 - \alpha + \beta\rho]\,/\alpha$, as shown above, as well as the terms in equation (13).

In particular,

Proposition 2 *If $N > \beta/(1-\beta)$, then (i) $r_P > r_C$ and as a result, $\left(x_1^C, x_2^C\right) > \left(\widehat{x}_1^C, \widehat{x}_2^C\right)$ while $\left(x_1^P, x_2^P\right) < \left(\widehat{x}_1^P, \widehat{x}_2^P\right)$; and (ii) both r_P/r_C and x_1^C/x_1^P (which equals x_2^C/x_2^P) increase with ρ. That is, if the capital bank is sufficiently connected, then (i) the credit of peripheral banks is more expensive than that of the bank at the center of the network, which causes the borrowers affiliated with the center-positioned bank to obtain too much resources and those with the peripheral banks too little, relative to the socially optimal allocation; and (ii) the higher is ρ, the worse are these issues and the lower is the efficiency attained by the equilibrium allocation.*

4 Interbank Insurance and Too-Connected-to-Fail

In this section, we focus on the case in which N is a large but finite number, especially $N > N^*$. Therefore, $\Upsilon_C > 0$, namely, the capital bank holds a net credit position to all the provincial banks. Again, we focus on the symmetric equilibrium in which the optimal decisions of the provincial banks are all the same.

As was said, we introduce risks to banks' asset side in this section. Specifically, we assume that at each region the entrepreneurs' productivity is a random variable which takes value $A > 0$ with probability $q > 1/2$ and value 0 with probability $1 - q > 0$. These risks are independent across banks. Considering that all the economic agents are risk neutral, to generate the demand for insurance, assume that it is costly for a bank to default on its promise to pay: the default cost is L_P to a provincial bank and NL_C to the capital bank; and L_P and L_C are both large, the exact meaning of which is explained later. Furthermore, we assume that all the insurance contracts are bilateral, a justification for which is that it is too costly to arrange multilateral contracts between banks. An insurance contract is thus represented by a profile of (C, μ), whereby the insurer bank is obliged to pay C to the insuree bank in the event of the latter receiving the negative shock and is paid with μC by the insuree if it receives the positive shock. Thus, C represents the coverage of the insuree bank, μ the insurance premium. Lastly, we assume that there is a fixed cost in arranging a contract (due to the time and effort it takes to settle the terms and conditions). The purpose that this assumption serves will be explained soon.

With the insurance introduced, the timing at $t = 0$ is changed as follows.

0A. Each bank posts the insurance premium μ that it will charge for selling insurance to other banks.

0B. Each bank decides first which banks to buy insurance from and then the insurance coverage C.

Afterwards, the rest of the timing is the same as in the baseline model: banks post the interest rate for the loans r; entrepreneurs decides the amount to borrow from their local banks m; they then move around to buy resources from households; households deposit their sales proceeds with their local banks; and banks net out the debts between them.

With the risks introduced, the timing at $t = 1$ is now as follows.

0. The shocks to banks' assets are realized and revealed.

1. Entrepreneurs settle their debts to the banks: those whose projects fail default and those whose projects succeed pay $m(1 + r)$ units of corn back.

2. Banks settle the interbank claims, of both credit and insurance, and redeem notes from households.

3. Agents consume.

The above timing of events at $t = 1$ suggests that interbank claims are senior to the claims of depositors. That is natural because in the model economy a bank that receives the negative shock need to obtain the insurance repayment before it can redeem notes from households. It also suggests that interbank credit claims are of the same seniority as the insurance claims, which allows netting between these two types of claims.

In the model economy, the insurance is done, ultimately, by pooling as many independent risks as possible. The presence of the fixed contracting cost prevents a single bank from pooling enough risks by contracting with a great many of other banks. As a result, risks are best shared by having a sole bank provide insurance to all the other banks. That is, the interbank insurance network should also be in a star structure. Theoretically, the bank at the center can be any bank, be it the capital bank or a provincial one. However, the equilibrium in which a provincial bank is positioned at the center being the sole insurance provider requires a great deal of coordination. Without it, any one that buys insurance from the provincial bank faces the risk that it is the only buyer of the bank, in which case it obtains not much insurance; indeed if it receives the negative shock it obtains no insurance payment at all with probability $1-q$ when the insurer bank itself receives the negative shock. In contrast, this problem is not there if the capital bank is at the center of the interbank *insurance* network, due to its position in the interbank *credit* network. The capital bank receives interbank credit repayments from all the provincial banks by Proposition 1 (as N is large). Hence on its asset side the risks have been *maximally* diversified. Even if there is only one bank that buys insurance from the capital bank, the buyer will obtain full insurance repayment unless in the extremely rare event that a large fraction of provincial banks default on their interbank credit payments to the capital bank. Considering the great difficulty of large scale coordination, we focus on the equilibrium in which the capital bank is the provider of insurance to all the provincial banks.

Passing on to analyzing this equilibrium, we make a final remark. On the equilibrium path, a provincial bank might still be able to pull banks away from trading insurance with the capital bank by offering a cheaper premium at stage 0A. Certainly, for a single bank considering deviation, this benefit of cheaper premium needs to be weighed against the cost of facing a higher probability of default, as discussed above. To simplify the exposition, we assume that the default cost L_P is large enough so that a provincial bank would rather buy insurance from the capital bank at its optimal, monopolistic, premium than deviate and be the sole buyer of insurance from a provincial bank at the break-even premium $\mu = 1 - q$.[12] As a result, in the equilibrium,

[12] The insuree pays μC with probability q, when it succeeds, while it obtains insurance repayment C with probability $(1-q)\,q$, when it fails and meanwhile insurer succeeds. Hence the break-even premium satisfies

the capital bank faces no meaningful competition in offering insurance. This assumption is not as restrictive as it might look because, as will be shown, the optimal monopolistic premium that the capital bank charges is low. Indeed, in some circumstances it is so low that the capital bank makes a loss from providing insurance, in which case the assumption is redundant because a provincial bank gains by buying insurance from the capital bank in itself, which thus dominates a deviation to buying insurance from another bank at the break-even price, with the consideration of default probability put aside.

The decisions of the agents depend on N. For any interesting variable x, it is more difficult to find its exact value $x(N)$ than to find its limit value when N goes to infinity, namely $\lim_{N\to\infty} x(N)$. In what follows, the latter is what we are concerned with, and we use notation $x(N) \approx y$ or even $x \approx y$ to mean $\lim_{N\to\infty} x(N) = y$. Doing so gains technical simplicity without incurring much loss because with N being large, $x(N)$ approximately equals y.

By backward induction, we start with entrepreneurs' decisions. Of them the analysis is almost the same as that in the preceding section except the complication here that at $t = 0$, banks' notes are discounted with a factor of $\nu < 1$ because in a certain probability banks default. Thus, if an entrepreneur borrows notes with face value m, their market value is $m\nu$. Entrepreneurs succeed with probability q. Their decision problem is now:

$$\max_{m,x_1,x_2} q \times \left\{ A \left(x_1^{\beta} x_2^{1-\beta} \right)^{\alpha} - m(1+r) \right\},$$

$$s.t.\ m\nu = p_1 x_1 + p_2 x_2.$$

which is equivalent to the problem:

$$\max_{m\nu,x_1,x_2} A \left(x_1^{\beta} x_2^{1-\beta} \right)^{\alpha} - (m\nu) \times \frac{1+r}{\nu},$$

$$s.t.\ m\nu = p_1 x_1 + p_2 x_2.$$

Namely, entrepreneurs' decision problem is equivalent to that in the preceding section, given by (1), apart from that the demand for notes is now $m\nu$ instead of m and the interest rate is $(1+r)/\nu$ instead of $1+r$. Substitute m with $m\nu$ and $1+r$ with $(1+r)/\nu$ in (2), and their

$-q \times \mu C + (1-q)\, qC = 0.$

demand for the local banks' notes satisfies: $m\nu = m((1+r)/\nu; p_1, p_2)$, which leads to the inverse demand function:

$$r = \nu (m\nu)^{-(1-\alpha)} \xi - 1 \tag{14}$$

where $\xi = A\alpha \left(\beta^{\beta}(1-\beta)^{1-\beta}\right)^{\alpha} \left(p_1^{\beta} p_2^{1-\beta}\right)^{-\alpha}$. Later we will show that $\nu \approx 1$ for any bank in equilibrium as N is a large number. As a result, all banks' notes are interchangeable on the one-to-one base, as was in the preceding section.

Now consider a representative provincial bank's decision on the scale of issuance M and the insurance coverage C, given that the capital bank charges insurance premium μ and issues M_C, and that all the other provincial banks choose $\left(\widetilde{M}, \widetilde{C}\right)$. Let $I(C,n)$ denotes the repayment of insurance in the state where n other provincial banks receive positive shocks. The provincial bank's balance sheet in state n is as follows:

Assets	Liabilities
Loans: { 0 with the bank receiving the negative shock $M(1+r)$ with the positive shock }	Equity: E
Insurance: { $I(C,n)$ with the negative shock $-\mu C$ with the positive shock }	Notes to redeem: D_P
	Interbank liabilities: $-\Upsilon_P \times (1+\rho)$

Table 3: The balance sheet of a representative bank with two types of interbank claims in state n

The total liability of the bank, denoted by Λ, is $\Lambda = D_P + [-\Upsilon_P \times (1+\rho)]$. The bank's expected value is then

$$\Pi(C,M) := q \times [M(1+r) - \mu C - \Lambda] + (1-q) \times \int_0^1 \max\left(I(C,n) - \Lambda, 0\right) dG(z) - Q_P(C,M)L_P, \tag{15}$$

where $Q_P(C,M) := (1-q)\Pr\left(I(C,n) < \Lambda\right)$ denotes the ex ante probability that the bank defaults. As banks' liabilities are inter-exchanged one-to-one, deposit D_P and interbank credit positions Υ_P are calculated as before, given by (4) and (5) respectively. It follows that $D_P \approx \beta\widetilde{M}$

and $\Upsilon_P \approx -M + \beta\widetilde{M}$. Therefore,

$$\Lambda \approx M + \left(M - \beta\widetilde{M}\right)\rho. \tag{16}$$

To find the contingent insurance repayment $I(C, n)$, we turn to the balance sheet of the capital bank in state n, which, with the choices of provincial banks given as above, is as follows:

Assets	Liabilities
Loans to entrepreneurs: π	Insurance liabilities: $(N-1-n)\times\widetilde{C}+C$
Insurance premium $n\times\mu\widetilde{C}$	Notes to redeem: D_C
Interbank credit: $\Upsilon_C\times(1+\rho)$	Equity: E

Table 4: The balance sheet of the capital bank in the above state n

On the asset side, with the aggregate lending of all the other banks $M' = (N-1)\widetilde{M} + M$, by (8),

$$\Upsilon_C = -\beta M_C + (1-\beta)\left((N-1)\widetilde{M} + M\right).^{13} \tag{17}$$

Then $\Upsilon_C \approx N \times (1-\beta)\widetilde{M}$. Let

$$z := \frac{n}{N}.$$

With the profit from loans π invariant with N, the value of the capital bank's assets is approximately equal to $N\times\left[z\mu\widetilde{C} + (1-\beta)(1+\rho)\widetilde{M}\right]$, its insurance liability to $N\times(1-z)\widetilde{C}$. Given that the interbank insurance liabilities are senior to the liabilities born on the notes, the capital bank does not defaults on the insurance repayments if and only if $z\mu\widetilde{C}+(1-\beta)(1+\rho)\widetilde{M} \geq (1-z)\widetilde{C}$, or equivalently $z \geq z_i$, where

$$z_i := \frac{\widetilde{C} - (1-\beta)(1+\rho)\widetilde{M}}{(1+\mu)\widetilde{C}}, \tag{18}$$

in which case $I(C, n) = C$. In the case of default, all revenue on its asset side is distributed to the insurees in proportion to the sizes of their claims. A unit of claim is repaid thus with $\left[z\mu\widetilde{C} + (1-\beta)(1+\rho)\widetilde{M}\right] / \left[(1-z)\widetilde{C}\right] < 1$ unit of corn. The representative provincial bank,

[13] Υ_C is independent of n because if a provincial bank receives the negative shock, its liability to the capital bank is still covered by the insurance repayment from the latter and thus no provincial banks default on their interbank liabilities to it.

by holding a claim of C, is thus repaid with $C \times \left[z\mu\widetilde{C} + (1-\beta)(1+\rho)\widetilde{M}\right] / \left[(1-z)\widetilde{C}\right]$. Two cases put together,

$$I(C, zN) = \min\left(1, \frac{z\mu\widetilde{C} + (1-\beta)(1+\rho)\widetilde{M}}{(1-z)\widetilde{C}}\right) C. \tag{19}$$

Let $\gamma := C/\Lambda \geq 1$ denote the insurance coverage per unit of liability. With this contingent insurance repayment, it is straightforward to find that the ex ante probability of default $Q_P = (1-q)\Pr(I(C,n) < \Lambda)$ as follows:

$$Q_P = \left\{ \begin{array}{c} 1-q \text{ if } \gamma < 1. \\ (1-q)G(z_P) \text{ otherwise} \end{array} \right\}, \tag{20}$$

where

$$z_P = \frac{\widetilde{C} - \gamma(1-\beta)(1+\rho)\widetilde{M}}{(1+\mu\gamma)\widetilde{C}} \tag{21}$$

and $G(\cdot)$ is the c.d.f. of $z = n/N$. Threshold of default z_P decreases with γ, naturally. At $\gamma = 1$, $z_P = z_i$ because if the coverage exactly suffices to clear the liability, then the bank defaults whenever it cannot obtain the full repayment of insurance, namely if $z < z_i$. Furthermore, as z_P depends only γ, so does Q_P, which can thus be written as $Q_P(\gamma)$. Observe that $Q_P(\gamma)$ jumps at $\gamma = 1$: $\lim_{\gamma \to 1-} Q_P(\gamma) = 1 - q > Q_P(1)$.

Instead of working with (C, M), it is more convenient to work with (Λ, γ), which connects with (C, M) via

$$M = \frac{1}{1+\rho}\left(\Lambda + \rho\beta\widetilde{M}\right) \tag{22}$$

$$C = \Lambda\gamma. \tag{23}$$

Substitute $I(C, zN)$ given by (19) into (15), rearrange, and the bank's problem becomes

$$\max_{\Lambda,\gamma} V(\Lambda,\gamma) := q \times [M(\Lambda)(1+r) - (1+\mu\gamma)\Lambda] + (1-q)\Lambda \times T(\gamma) - Q_P(\gamma)L_P, \tag{24}$$

with

$$T(\gamma) := \left\{ \begin{array}{c} 0 \text{ if } \gamma < 1 \\ \left[\int_{z_i}^{1} (\gamma - 1)\, dG(z) + \int_{z_P(\gamma)}^{z_i} \left(\frac{z\mu\widetilde{C} + (1-\beta)(1+\rho)\widetilde{M}}{(1-z)\widetilde{C}}\gamma - 1\right) dG(z)\right] \text{ if } \gamma \geq 1 \end{array} \right\}, \tag{25}$$

subject to the constraints (14), which connects r to M, and that

$$M(1+r) - (1+\mu\gamma)\Lambda \geq 0, \tag{26}$$

which states that the provincial bank will not default on receiving the positive shock. This constraint should, intuitively, never be binding, but, surprisingly, it can be binding, as will be shown.

Let the $\left(C\left(\mu;\widetilde{C},\widetilde{M}\right),M\left(\mu;\widetilde{C},\widetilde{M}\right)\right)$ denote the optimal choice of the bank. The symmetric equilibrium in which all the provincial banks make the same choice satisfies $\left(C\left(\mu;\widetilde{C},\widetilde{M}\right),M\left(\mu;\widetilde{C},\widetilde{M}\right)\right)=\left(\widetilde{C},\widetilde{M}\right)$. Let $\left(C\left(\mu\right),M\left(\mu\right)\right)$ denote the choice of provincial banks in this equilibrium given the insurance premium μ.

Consider now the decision problem of the capital bank, whose balance sheet is presented in Table 4, with $\left(\widetilde{C},\widetilde{M}\right)=\left(C,M\right)=\left(C\left(\mu\right),M\left(\mu\right)\right)$. As before, the funds deposited in equal the value of type 2 resources and thus $1-\beta$ fraction of the aggregate lending, that is,

$$D_C=\left(1-\beta\right)\left(\left(NM\left(\mu\right)+M_C\right).\right.$$

According to the balance sheet, thus, the capital bank's profit in state n is $V_n:=\left[\pi+n\times\mu C\left(\mu\right)+\Upsilon_C\times\left(1+\rho\right)\right]-\left[\left(N-n\right)\times C\left(\mu\right)+\left(1-\beta\right)\left(NM\left(\mu\right)+M_C\right)\right]$, if $V_n\geq 0$, otherwise, it defaults and suffers a loss of NL_C. As the revenue from loans $\pi=qM_C\left(1+r_C\right)$ and Υ_C given by (17) and recall $z=n/N$, we have

$$V_n=\left[qM_C\left(1+r_C\right)-\left(1+\beta\rho\right)M_C\right]+N\times\left\{\left[z\mu-\left(1-z\right)\right]C\left(\mu\right)+\left(1-\beta\right)M\left(\mu\right)\rho\right\},$$

where the first term represents the profit from lending notes, for which the marginal cost is $1+\beta\rho$ because β fraction of notes flows out become interbank liabilities charged with interest ρ; and the second term represents the profit from the bank's insurance business and from its interbank credit position. With N being large, $V_n/N\approx\left[z\mu-\left(1-z\right)\right]C\left(\mu\right)+\left(1-\beta\right)M\left(\mu\right)\rho$ and hence $V_n/N<0$, namely the capital bank defaults, if and only if $\left[z\mu-\left(1-z\right)\right]C\left(\mu\right)+\left(1-\beta\right)M\left(\mu\right)\rho<0$, or equivalently $z<z_C$, where

$$z_C\left(\mu\right):=\frac{C\left(\mu\right)-\left(1-\beta\right)\rho M\left(\mu\right)}{(1+\mu)C\left(\mu\right)}. \tag{27}$$

Therefore, the capital bank defaults with probability $G\left(z_C\left(\mu\right)\right)$. By choosing $\left(\mu,M_c\right)$, the bank's value is thus $E_n\left(\max\left(V_n,0\right)\right)-G\left(z_C\right)\times NL_C$. With V_n given above, the capital bank's problem is

$$\begin{aligned}\max_{\mu,M_C}U(\mu,M_C)\quad:&\quad=M_C\left[q\left(1+r_C\right)-\left(1+\beta\rho\right)\right]\left[1-G\left(z_C\left(\mu\right)\right)\right]+N\times\left[\phi(\mu;\rho)-G\left(z_C\left(\mu\right)\right)L_c\right],\\&\quad s.t.(14)\text{ (which connects } r_C \text{ with } M_c),\end{aligned}$$

where

$$\phi(\mu;\rho) := \int_{z_{C(\mu)}}^{1} \left\{ \left[z\mu - (1-z) \right] C(\mu) + (1-\beta)\rho M(\mu) \right\} dG(z). \tag{28}$$

Obviously, the decision on M_C is determined by the following problem

$$\max_{M_C} M_C \left[q(1+r_C) - (1+\beta\rho) \right], \text{ s.t. } (14).$$

The value of this problem – which is the capital bank's profit from lending – stays invariant with N. With N being large, therefore, this profit is vanishingly small relative to its profit from its interbank credit and insurance positions, i.e. the second term of $U(\mu, M_C)$. It follows that the decision on μ is independent of the profit from loans and approximately determined by

$$\max_{\mu} \phi(\mu;\rho) - G(z_C(\mu)) L_c. \tag{29}$$

Instead of μ, it is actually more convenient to work with

$$\upsilon := q\mu - (1-q).$$

This υ can be regarded as the profit margin to the capital bank in providing insurance: with a unit of coverage, with probability q the insuree bank sees its loans performing and thus pays μ to the capital bank, while with probability $1-q$, the capital bank is obliged to pay one unit to the insuree bank.

To find out more about these objective functions, observe that by the Central Limit Theorem, the distribution of $(n - Nq)/\sqrt{Nq(1-q)}$ converges to the Standard Normal distribution, $N(0,1)$, with c.d.f. $\Phi(\cdot)$. With $z = n/N$, $(n-Nq)/\sqrt{Nq(1-q)} = (z-q)\sqrt{N}/\sqrt{q(1-q)}$. It follows that $G(z) \approx \Phi\left(\frac{(z-q)\sqrt{N}}{\sqrt{q(1-q)}}\right)$.

Lemma 1 *In equilibrium $z_C < q$.*

Proof. *See Appendix.* ■

By this lemma, If $N \to \infty$, the probability that the capital bank defaults, $G(z_C) \approx \Phi\left(\frac{(z_C-q)\sqrt{N}}{\sqrt{q(1-q)}}\right)$, converges to zero in the order of e^{-xN}, for some positive x. Furthermore, if $\gamma \geq 1$,then $z_P < z_C$;[14] intuitively, if $z \geq z_C$, then the capital bank does not default on

[14] By (21) and (27), with $\left(\widetilde{C}, \widetilde{M}\right) = (C(\mu), M(\mu))$, $z_C > z_P \Leftrightarrow \frac{C(\mu)-(1-\beta)\rho M(\mu)}{(1+\mu)C(\mu)} > \frac{C(\mu)-\gamma(1-\beta)(1+\rho)M(\mu)}{(1+\mu)C(\mu)}$, which obviously holds true if $\gamma \geq 1$.

the insurance payment and if this payment suffices to cover a provincial bank's liabilities (i.e. $\gamma \geq 1$), then it will not default, that is, $z \geq z_P$. Hence, if a provincial bank chooses $\gamma \geq 1$, then it default probability converges to zero in the order of $e^{-x'N}$, for some $x' > x$. We saw in (20) that if it chooses $\gamma < 1$, then the probability of default is $1-q$. As we have assumed that the cost of default L_P is large enough, the provincial banks all choose $\gamma \geq 1$,[15] which, hereafter, will be added as a constraint to provincial banks' problem. Consequently, the lemma ensures that all banks' probabilities of default are vanishingly small. This result is intuitive: Given that there is a large number of independent risks to be pooled together and that the cost of default is high, banks are able to make insurance arrangement to reduce the risk of default to a negligible level. The probability of bank default being vanishingly small probability leads to $\nu \approx 1$ *for all banks. That is, all banks' notes are approximately worth the face values and hence interchangeable one to one*, as was said.

It also leads us to find the value of the integration in the objective function of provincial banks and that of the capital bank, given respectively in (25) and (28), as follows.

Lemma 2 *(i) for* $\gamma \geq 1$, $T(\gamma) \approx \gamma - 1$ *and* $T'(\gamma) \approx 1$. *(ii):* $\phi \approx \upsilon C(\upsilon) + (1-\beta)\rho M(\upsilon)$ *and* $\phi'_\upsilon \approx \upsilon C'(\upsilon) + C(\upsilon) + (1-\beta)\rho M'(\upsilon)$.

Proof. See Appendix. ■

Intuitively, the lemma says that the integrations approximately take the value with the lower limit being 0 instead of a threshold of z. For example, $\phi \approx \int_0^1 \{[\mu z - (1-z)]C + (1-\beta)\rho M\} dG(z) = [\mu q - (1-q)]C + (1-\beta)\rho M = \upsilon C + (1-\beta)\rho M$ as $\int_0^1 z dG(z) = q$.

Furthermore, because default probabilities $G(z_C)$ and $G(z_P)$ converge to zero with $N \to \infty$ in the order of e^{-xN} for some $x > 0$, $G'(z_C)$ and $G'(z_P)$ also converge to zero, in the order of Ne^{-xN}. Hence, given the default costs L_P and L_C, the effect of banks' choices on the expected default costs is negligible.

[15] A sufficient condition for $\gamma \geq 1$ is to let a provincial bank's value at $\gamma = 0$ dominated by that at $\gamma = 1$, that $\max_\Lambda V(\Lambda, 0) < \max_\Lambda V(\Lambda, 1)$. That, with $V(\Lambda, \gamma)$ given in (24), follows from $\max_\Lambda \{q \times [M(\Lambda)(1+r) - \Lambda]\} - (1-q)L_P < \max_\Lambda \{q \times [M(\Lambda)(1+r) - (1+\mu)\Lambda]\} - (1-q)G(z_P)L_P$, or equivalently $(1-q)(1-G(z_P))L_P > \max_\Lambda \{q \times [M(\Lambda)(1+r) - \Lambda]\} - \max_\Lambda \{q \times [M(\Lambda)(1+r) - (1+\mu)\Lambda]\}$.

This observation and Lemma 2 together, the representative provincial bank's problem, given in (24), becomes:

$$\max_{\Lambda,\gamma} V(\Lambda,\gamma) := q \times [M(\Lambda)(1+r) - (1+\mu\gamma)\Lambda] + (1-q)\Lambda \times (\gamma - 1), s.t. \tag{30}$$

$$(14); \quad M(\Lambda)(1+r) - (1+\mu\gamma)\Lambda \geq 0; \quad \gamma \geq 1,$$

where $M(\Lambda) = \frac{1}{1+\rho}\left(\Lambda + \rho\beta\widetilde{M}\right)$ is given by (22). And the capital bank's problem on deciding μ or rather $\upsilon = q\mu - (1-q)$, given in (29), becomes:

$$\max_{\upsilon} \Pi(\upsilon,\rho) := \upsilon C(\upsilon) + (1-\beta)\rho M(\upsilon). \tag{31}$$

With the first problem solved, provincial banks' choice is summarized in the following lemma.

Lemma 3 *Given υ, provincial banks choose money issuance of scale $M(\upsilon) = \left(\frac{q\xi\alpha}{(1+\rho)(1+\upsilon)}\right)^{\frac{1}{1-\alpha}}$ and insurance of coverage $C(\upsilon) = \chi(\upsilon)M(\upsilon)$, where $\chi(\upsilon) = \left\{\begin{array}{c} 1+(1-\beta)\rho \text{ if } \upsilon > 0 \\ \frac{(1+\rho)(1+\upsilon)-\alpha q(1+(1-\beta)\rho)}{\alpha(1-q+\upsilon)} \text{ if } \upsilon < 0 \end{array}\right\}$, which gives rise to coverage per unit of liability $\gamma(\upsilon) = \left\{\begin{array}{c} 1 \text{ if } \upsilon > 0 \\ 1 + \frac{(1+\upsilon)[1-\alpha+\rho(1-\alpha+\alpha\beta)]}{\alpha(1-q+\upsilon)(1+(1-\beta)\rho)} \text{ if } \upsilon < 0 \end{array}\right\}$.*

Proof. See Appendix. ■

Observe that provincial banks' scale of issuance is a continuous function of υ at $\upsilon = 0$, but their demand for insurance jumps by a scale of $\lim_{\upsilon\to 0-}\gamma(\upsilon) - \lim_{\upsilon\to 0+}\gamma(\upsilon) = \frac{1-\alpha+(1-\alpha+\alpha\beta)\rho}{\alpha(1-q)(1+(1-\beta)\rho)}$ per unit of liability at $\upsilon = 0$, that is, at $\mu = (1-q)/q$ as $\upsilon = q\mu - (1-q)$.

Having found $(M(\upsilon), C(\upsilon))$ in the above lemma, we can solve the capital bank's problem given in (31) and find its optimal choice of insurance premium has the following property.

Proposition 3 *the optimal insurance premium μ^* decreases with the interbank rate ρ.*

Proof. See Appendix. ■

An intuition for the proposition is as follows. The capital bank can derive profit from both types of interbank claims – credit and insurance. However, these two channels are in a conflict. To obtain more profit from the interbank credit positions, the capital bank wants to encourage provincial banks to increase lending. The scale of lending decreases with the premium of insurance, as $M'(\upsilon) < 0$. Intuitively, when provincial banks lend out their liability, they need to buy insurance to cover it in the event of receiving negative shocks. As a result, the cost of insurance υ partakes the cost of lending and a higher insurance cost leads to a smaller scale of lending. Therefore, to encourage provincial banks to increase lending, the capital bank has to lower the insurance premium, obtaining less profit from selling insurance. Moreover, the higher the interbank rate, the greater the benefit the capital bank obtains from its interbank credit positions, to increase which it chooses a lower insurance premium, hence the proposition.

According to the proposition, if ρ goes up, then μ^* goes down. But how low can it go? It looks that it should never be so low that the capital bank makes a loss in providing insurance, that is, $\upsilon^* < 0$. Counter-intuitively, that happens if ρ is high enough. To show this, consider under which conditions indeed the capital bank chooses $\upsilon > 0$, the negation of which defines the circumstances where it chooses $\upsilon \leq 0$. Substitute $C(\upsilon)$ and $M(\upsilon)$ in Lemma 3 above for $\upsilon > 0$ into $\Pi(\upsilon, \rho)$ given in (31), and we find $\Pi'_\upsilon = 0$ leads to

$$\upsilon^* = \frac{1}{\alpha\left(1 + (1-\beta)\,\rho\right)} - 1.$$

This $\upsilon^* > 0$ if and only if $\rho < \frac{1-\alpha}{\alpha(1-\beta)}$. If $\rho > \frac{1-\alpha}{\alpha(1-\beta)}$, therefore, supposing an optimal choice of $\upsilon > 0$ is self-contradictory. This result is restated in the following proposition.

Proposition 4 *If $\rho > \frac{1-\alpha}{\alpha(1-\beta)}$, then $\upsilon^* \leq 0$, that is, the capital bank loses value from providing insurance to provincial banks.*

By Lemma 3 and the ensuing discussion, $C(\upsilon)$ is decreasing with υ and jumps at $\upsilon = 0$. By Proposition 3 and the ensuing discussion, υ decreases with ρ and $\upsilon = 0$ at $\rho = \frac{1-\alpha}{\alpha(1-\beta)}$. These two observation together imply that *the insurance coverage that provincial banks choose* (i.e. C) *increases with the interbank interest rate* (i.e. ρ) *and jumps at* $\rho = \frac{1-\alpha}{\alpha(1-\beta)}$.

Thus far, we have investigated the interplay between the two types of interbank claims – of credit and insurance. One effect of this interplay sees the capital bank to become the sole provider of insurance to all the provincial banks. Then, the capital bank should be too connected to fail:

its failure means no insurance to all the provincial banks, which might inflict great stress on to them or even bring them all down. If that happens, it would be an event of system meltdown. To explore this intuition, and to endogenize the event of system meltdown, the present setting is modified and further extended in the following section.

5 The Risk of Systemic Meltdown and Its Early Warning

In this section, we assume that there is a period $t = 1/2$, between $t = 0$ and $t = 1$, and that during $t = 1/2$, the shocks to provincial banks, namely, $\widetilde{A} = A$ or 0, are revealed *sequentially* in a queue.[16] To keep the symmetry between provincial banks, assume that ex ante each and every provincial bank has an equal chance to be at any position of this queue. Furthermore,

Assumption K2: In order for their liabilities to be accepted as a means of payment, banks give depositors right to convert a deposit of face value F into $(1-\delta)F$ units of corn at any moment of $t = 1/2$, with $0 < 1-\delta < 1$.

This fraction $1-\delta$ is exogenous. Essentially, $\frac{1}{1-\delta} - 1$ is the net interest rate to depositing over the time from $t = 1/2$ to $t = 1$. As in real life this interest rate is almost zero, we shall expect δ is close to 0. In particular, we assume that

$$0 < \delta < q(1-q). \tag{32}$$

The assumption captures the real life observation that the most common form of banks' liabilities that are used as a means of payment is demand deposit, which entails the right to withdraw at demand. To make this right meaningful in the model economy, we assume in this section that at $t = 0$, banks are endowed with $G \approx 0$ units of corn, which is stored over time and is used to meet the demand of withdraw, on the first-come-first-serve base. If a bank still faces outstanding demands of withdraw when its liquid asset, namely its corn stock, has been depleted, it suspends redemption. This assumption abstracts away the issue of mis-coordination induced bank run and simplifies the analysis of depositors' decision on whether to withdraw at

[16] Given N is large and the capital bank's profit from loans is negligible, the timing of its shock being revealed does not matter.

$t = 1/2$. To simplify this analysis further, we assume that if a bank defaults, the value of all its deposits is annihilated, partly due to the high cost of bankruptcy.

By this assumption, if a depositor with a representative provincial bank of face value F holds on to $t = 1$, he obtains the expected payoff of $(1 - Q) F$, where Q is the probability that the bank will default at $t = 1$, evaluated at $t = 1/2$. This default probability Q depends on ϵ, the fraction of deposits that have been withdrawn at $t = 1/2$, that is, $Q = Q(\epsilon)$. To see this, consider the bank's balance sheet, which is as given in Table 3, with the addition of the corn stock on the asset side. If ϵ fraction of deposits are withdrawn at $t = 1/2$, the bank's total liability decreases by ϵD_P, while its corn stock decreases by $(1 - \delta) \times \epsilon D_P$. The bank defaults in the event of receiving the negative shock, then, if and only if $[G - \epsilon (1 - \delta) D_P] + I(C, z) < (1 - \epsilon) D_P + [-\Upsilon_P (1 + \rho)]$, or equivalently:

$$I(C, z) < -\epsilon\delta D_P + D_P + [-\Upsilon_P (1 + \rho)] - G. \tag{33}$$

This is equivalent to $z < z_P(\epsilon; G)$ for some threshold which depends on ϵ and G. Thus probability of default $Q = G(z_P(\epsilon; G))$ if the bank is revealed to have received the negative shock and $Q = (1 - q) G(z_P(\epsilon; G))$ if it is not. Observe that the right hand side of inequality (33) decreases with ϵ. Therefore, $z_P(\epsilon; G)$ decreases with ϵ. Hence $Q'(\epsilon) < 0$. In particular, $Q(0) > Q(\bar{\epsilon})$, where $\bar{\epsilon}$ is the maximum fraction of deposits that can be withdrawn, determined by the constraint that the bank has G units of corn during $t = 1/2$ to meet the withdraw demands, that is, $\epsilon (1 - \delta) D_P \leq G$. Hence,

$$\bar{\epsilon} := \frac{G}{(1 - \delta) D_P}. \tag{34}$$

Lemma 4 *During period $t = 1/2$, a bank's depositors run to the bank demanding withdraw if $Q(\bar{\epsilon}) > \delta$ and they stay put if $Q(0) < \delta$.*

Proof. If $Q(\bar{\epsilon}) > \delta$, then $Q(\epsilon) > \delta$ for any feasible ϵ because $Q'(\epsilon) < 0$. It means that $(1 - \delta) F > (1 - Q(\epsilon)) F$ for any ϵ and F. Therefore, no matter what other depositors do, represented by a value of ϵ, a depositor with any size of claims F is better off to withdraw all his claims at $t = 1/2$, if he can, to obtain $(1 - \delta) F$, than to hold them to $t = 1$. He is able to withdraw his claims only if he gets to the bank before its liquid asset is depleted for satisfying earlier demands of withdraw. Therefore, all depositors will endeavour to get to the bank early enough, namely a bank run occurs.

If $Q(0) < \delta$, then similarly $(1-\delta)F < (1-Q(\epsilon))F$ for any ϵ and F. Therefore, no matter what other depositors do, namely whatever is ϵ, a depositor is better off to hold his claims to $t=1$ than to withdraw at $t=1/2$. Therefore, he stays put. ∎

We have assumed that $G \approx 0$, which implies $\overline{\epsilon} \approx 0$ by (34). Therefore, $z_P(\epsilon; G) \approx z_P(0;0) = z_P$, the threshold in the preceding section. Hence, $Q(0) \approx Q(\overline{\epsilon}) \approx (1-q)\Pr(z < z_P)$ if the bank's shock has not been revealed. Intuitively, the assumption of $G \approx 0$ saves us from the complication of considering the case in which $Q(0) > \delta > Q(\overline{\epsilon})$.

Consider now the possibility of run to a large fraction of banks. As banks' shocks are revealed sequentially, at any moment in period 1/2, the information set of depositors consists of the number of the banks revealed to have received a positive shock, S, and the number of banks revealed to have received a negative shock, F, while for the rest $N-S-F$ banks, the shocks are still unknown. Let $f := F/N$ and $s := S/N$. At any moment, the information publicly observed is thus (f,s). Conditional on it, banks revealed to have received a negative shock default with probability $\Pr(z < z_P|(f,s))$, and banks whose shocks are un-revealed default with probability $(1-q)\Pr(z<z_P|(f,s))$. If $(1-q)\Pr(z<z_P|(f,s)) > \delta$, by Lemma 4, bank run occurs to all banks except those revealed to have positive shocks, whose assets are known to be sound with certainty, that is, it occurs to fraction $1-s$ of banks. If this event occurs, as we will see, typically s is small. Thus we define this event as *the event of the banking system meltdown* and define the *event zone* as

$$\Omega := \left\{(f,s) \,|\, \Pr(z<z_P|(f,s)) > \frac{\delta}{1-q}\right\}.$$

Given (f,s), the final fraction of positive shocks z satisfies $s \leq z \leq 1-f$. Therefore,

$$\begin{aligned} \Pr(z<z_P|(f,s)) &= \Pr(z<z_P|s\leq z\leq 1-f) \\ &= \left\{\begin{array}{c} 0 \text{ if } s \geq z_P \\ \frac{G(z_P)-G(s)}{G(1-f)-G(s)} \text{ if } s < z_P \end{array}\right\}. \end{aligned} \tag{35}$$

To characterize the event zone, define $f := \Gamma(s)$ as the function implicitly defined by

$$\frac{G(z_P)-G(s)}{G(1-f)-G(s)} = \frac{\delta}{1-q} := \widetilde{\delta} \tag{36}$$

for $s \leq z_P$. Then $\underline{f} := \Gamma(0) \in (0, 1 - z_P)$,[17] $\Gamma(s) < 1 - s$ for $s < z_P$ (as $G(1 - \Gamma(s)) - G(s) > 0$), and at $s = z_P$, $\Gamma(s) = 1 - z_P$. Moreover,

Lemma 5 *$\Gamma'(s) > 0$. At $s = 0$, $\Gamma'(s) \approx 0$. And at $s = z_P$, $\Gamma'(s) = \frac{1-\widetilde{\delta}}{\widetilde{\delta}}$.*

Proof. See Appendix. ■

As δ is small, hence so is $\widetilde{\delta}$. Therefore at the end point, namely, at $s = z_P$, $\Gamma'(s)$ is large. By (35), $\Pr(z < z_P | (f, s))$ increases with f. Therefore, $(f, s) \in \Omega$ if and only if $f > \Gamma(s)$, that is, the systemic event zone is:

$$\Omega := \{(f, s) \,|\, f > \Gamma(s), s \in [0, z_p]\}.$$

Each sequence of revelations of provincial banks' shocks is represented by a pair of functions $(f(z), s(z))$ over $z \in [0, 1]$ that satisfies $(f(0), s(0)) = (0, 0)$, $(f'(z), s'(z)) \geq (0, 0)$, and $f(z) + s(z) = z$. That is, it is represented by a path in the triangle $\{(f, s) | f, s \geq 0 \text{ and } f + s \leq 1\}$ that starts from $(0, 0)$, always advances in the directions of north-east quarter, and ends onto the boundary line defined by $f + s = 1$. Then $s(1)$ represents the ultimate average quality of banks' assets. If a path enters the event zone, that is, if $(f(z), s(z)) \in \Omega$ for some $z \in [0, 1]$, then the system melts down on the way. However, according to the Central Limit Theorem, ex ante 99% of the paths occur $3.29\sigma = 3.29 \times \sqrt{\frac{q(1-q)}{N}}$ away from the straight path defined by $f/s = (1 - q)/q$, which is thus called as the *major path*. The next lemma shows that the curve $f = \Gamma(s)$ lies above the major event path, therefore, the event of system meltdown is a rare event indeed.

Lemma 6 *$\Gamma(s) > (1 - q)/q \times s$ for $s \leq z_P$.*

Proof. See Appendix. ■

These two lemmas brings about the following illustration of the event zone:

[17] $\underline{f} < 1 - z_P \Leftrightarrow 1 - \underline{f} > z_P \Leftrightarrow G\left(1 - \underline{f}\right) > G(z_P) \Leftrightarrow G(z_P)/\widetilde{\delta} > G(z_P) \Leftrightarrow \delta < 1 - q$, which is assumed. $\underline{f} > 0$ because if N is large enough then $G(z_P) < \widetilde{\delta}$. It follows that $G\left(1 - \underline{f}\right) = G(z_P)/\widetilde{\delta} < 1$, that is, $\underline{f} > 0$.

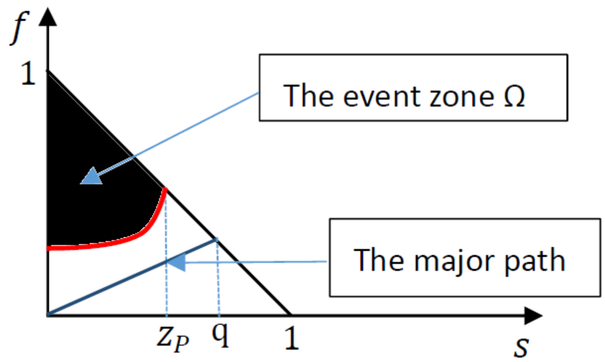

Figure 1: If the present information (f, s) is in the event zone Ω, the banking system melts down, which, however, happens rarely.

Two observations follow from the figure above. First, given the overall outcome $s(1)$, if $s(1) \in (z_P, 1 - \underline{f})$, whether the system meltdown occurs depends on the path along which the states of provincial banks are revealed. Intuitively, if good news – namely, positive shocks – are front-loaded, then depositors' estimation about the overall outcome will be good enough to keep them staying put all the way. However, if negative shocks are front-loaded, their estimation about the overall outcome will be bad enough to trigger a bank run. That is because the shocks are independent and ex ante they cannot expect that un-revealed shocks will be positive in such a skewed proportion as to sufficiently offset the bad news that they have received. Both scenarios are illustrated in the figure below.

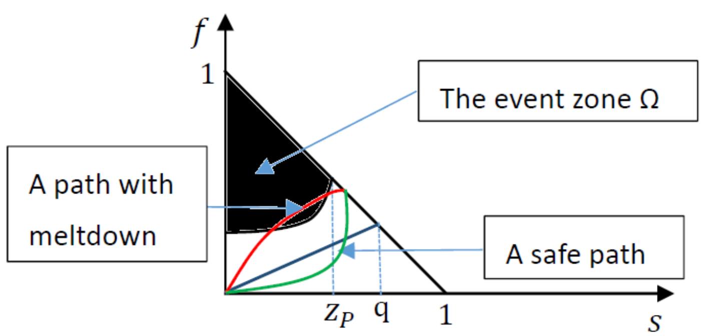

Figure 2: Path dependence: the upper red path and the lower green path leads to the same final outcome, but the system melts down when the upper path enters the event zone, whereas nothing happens along the lower path.

Mathematically,

Proposition 5 *If $s(1) < z_P$, all paths enter the event zone Ω. If $s(1) \geq 1 - \underline{f}$, no paths enter Ω. If $s(1) \in (z_P, 1 - \underline{f})$, some paths do, some not.*

Proof. See Appendix. ■

Second, as the shocks are independent, conditional on what has been revealed, (f, s), the future realization of un-revealed shocks concentrates along the path starting from the present state (f, s) and with slope $(1 - q)/q$. Hence, if $(f, s) \notin \Omega$ and this path enters Ω, then it is very likely that the system will develop from the present state (f, s) into the event zone, that is, it is very likely that system meltdown will happen in the future. Therefore, any such (f, s) is an early warning to the systemic risk. All such points of (f, s) form an early warning zone in which the banking system is still calm, but is very likely to melt down at some point of the future, whereby the government has a space of time to act. This early warning zone is illustrated as follows.

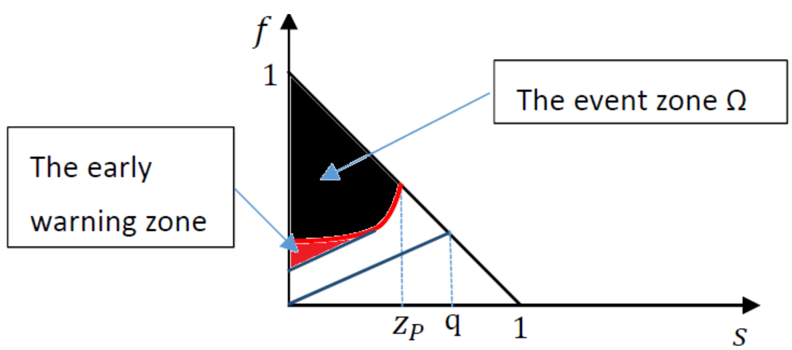

Figure 3: The early warning zone: If the system is presently in the early warning zone, the system is still calm but it is very likely to melt down.

Mathematically, the early warning zone is defined as:

$$\Phi := \left\{ (f, s) \,|\, f \leq \Gamma(s) \text{ and } f + \frac{1-q}{q}\tau > \Gamma(s + \tau) \text{ for some } \tau > 0 \right\}.$$

Proposition 6 *The early warning zone Φ is non-empty.*

Proof. See Appendix. ■

6 Conclusion

This paper presents a new approach to endogenize interbank credit networks, based on the specialty of commercial banks that their liabilities are widely accepted as a means of payment. By endogenizing a star-structured network, we apply this approach to study the issues of too connected to fail and systemic stability. We find that the bank at the center offers cheaper credit than the peripheral banks and the difference is greater if the interbank interest rate is higher, which causes resources inefficiently concentrated at the center. Moreover, if the bank at the center – namely the capital bank – is well connected, then it owes to all the peripheral banks. Therefore, the endogenous network of interbank credit has no issue of too connected to fail (through the effects on the asset side). However, the capital bank's position in this network gives it an advantage to provide all the peripheral banks with insurance against their idiosyncratic risks. In the presence of interbank insurance, therefore, the capital bank is too connected to fail. Its default on the insurance obligations triggers bank run to all the insuree banks unless their assets have been publicly observed to be sound. We find that whether this event happens is path dependent and early news matters more for systemic stability than late one; and that there exists an early warning zone of news based on which the event of system meltdown is likely to happen but has not happened yet, whereby the government has a space of time to take measures.

There are limitations with the present study of this paper regarding systemic risk. For example, it does not consider banks' decision on risk taking. Due to this limitation, banks' risks in the model economy are independent and identical, whereby the Normal distribution rules. Hence, the event of system meltdown happens only if an usually great fraction of banks receive negative shocks. That is, it is an event outside the three-sigma limits, whereas in reality that event seems to follow a fat-tailed distribution. This fact suggests that banks' assets be correlated. To some degree, this correlation results from decisions made by the banks, as demonstrated by Acharya (2009) and Acharya and Yorulmazer (2007, 2008). Incorporating these decisions into the framework of the present paper might give rise to a deeper investigation into the issue of systemic risk.

Appendix

The proof of Lemma 2:

By (25), for $\gamma \geq 1$,

$$T(\gamma) - (\gamma - 1) = -\left[\gamma \int_{z_P}^{z_i} \left(1 - \frac{z\mu\widetilde{C} + (1-\beta)(1+\rho)\widetilde{M}}{(1-z)\widetilde{C}}\right) dG(z) + (\gamma - 1) G(z_P)\right].$$

Thus $|T(\gamma) - (\gamma - 1)| < \gamma [G(z_i) - G(z_P)] + (\gamma - 1) G(z_P) = \gamma G(z_i) - G(z_P) < \gamma G(z_i) < \gamma G(z_C)|_{\text{Lemma 1}} \to 0$ if $N \to \infty$. Hence, $T(\gamma) \approx \gamma - 1$. Moreover,

$$\begin{aligned} T'(\gamma) &= 1 - G(z_i) + \int_{z_P(\gamma)}^{z_i} \frac{z\mu\widetilde{C} + (1-\beta)(1+\rho)\widetilde{M}}{(1-z)\widetilde{C}} dG(z) - \frac{dz_P(\gamma)}{d\gamma} \times \left(\frac{z\mu\widetilde{C} + (1-\beta)(1+\rho)\widetilde{M}}{(1-z)\widetilde{C}} \times \gamma - 1\right)_{z=z_P} \\ &= 1 - G(z_i) + \int_{z_P(\gamma)}^{z_i} \frac{z\mu\widetilde{C} + (1-\beta)(1+\rho)\widetilde{M}}{(1-z)\widetilde{C}} dG(z), \end{aligned}$$

because at $z = z_P$, $\frac{z\mu\widetilde{C}+(1-\beta)(1+\rho)\widetilde{M}}{(1-z)\widetilde{C}} \times \gamma - 1 = 0$. Hence, $|T'(\gamma) - 1| < G(z_i) + \int_{z_P(\gamma)}^{z_i} 1 dG(z) < 2G(z_i) < 2G(z_C)|_{\text{Lemma 1}} \to 0$ if $N \to \infty$. Hence, $T'(\gamma) \approx 1$.

By (28), $|\,\phi - (\upsilon C(\upsilon) + (1-\beta)\rho M(\upsilon))\,| = |\int_0^{z_C} \{\mu z C(\upsilon) - (1-z) C(\upsilon) + (1-\beta)\rho M(\upsilon)\} dG(z)| < \max_{0 \leq z \leq 1} \{\mu z C(\upsilon) - (1-z) C(\upsilon) + (1-\beta)\rho M(\upsilon)\} \times G(z_C) \to 0$ with $N \to \infty$. Moreover, as the integrand equals zero at $z = z_C$ by the definition of z_C, and $M\mu/M\upsilon = 1/q$,

$$\phi'(\upsilon) = \int_{z_C(\upsilon)}^{1} \left\{\frac{1}{q} z C(\upsilon) + (\mu z - (1-z)) C'(\upsilon) + (1-\beta)\rho M'(\upsilon)\right\} dG(z).$$

Hence, $|\phi'(\upsilon) - \{C(\upsilon) + \upsilon C'(\upsilon) + (1-\beta)\rho M'(\upsilon)\}| = |\int_0^{z_C} \left\{\frac{1}{q} z C(\upsilon) + (\mu z - (1-z)) C'(\upsilon) + (1-\beta)\rho M'(\upsilon)\right\} dG(z$ $\max_{0 \leq z \leq 1} \left\{\left\{\frac{1}{q} z C(\upsilon) + (\mu z - (1-z)) C'(\upsilon) + (1-\beta)\rho M'(\upsilon)\right\}\right\} \times G(z_C) \to 0$ with $N \to \infty$. Q.E.D.

The Proof of Lemma 3:

Consider the representative provincial bank's problem given in (30). With $M(\Lambda) = \frac{1}{1+\rho}\left(\Lambda + \rho\beta\widetilde{M}\right)$ by (22) and moreover

$$r = M^{-(1-\alpha)}\xi - 1. \tag{37}$$

due to $\nu \approx 1$ and (14), we find the first order derivatives of the objective function are $\frac{\partial V}{\partial \Lambda} = \frac{q\alpha\xi}{1+\rho}(M)^{-(1-\alpha)} - (1 + \upsilon\gamma)$; and $\frac{\partial V}{\partial \gamma} = -\upsilon\Lambda$, which, intuitively, is because to increase γ, the coverage per unit of liability, by one unit, the bank needs to buy Λ units of insurance, each of which costs υ. What follows depends on the sign of υ. Consider first the case in which $\upsilon > 0$. In this case, provincial banks lose value to the capital bank from buying the insurance. The only source of their profit is the loans, if and when they perform. Therefore, nonbinding is constraint (26), which commands that provincial banks shall obtain a non-negative profit in the event of their loans performing. With the presence of constraint $\gamma \geq 1$, the first order Kuhn-Tucker conditions of provincial banks' problem are

$$\frac{q\alpha\xi}{1+\rho}(M)^{-(1-\alpha)} - (1+\upsilon\gamma) = 0. \tag{38}$$

$$-\upsilon\Lambda \leq 0, \tag{39}$$

with the equality holds at $\gamma > 1$. Given $\upsilon > 0$, the equality never holds and hence at the optimum $\gamma = 1$, namely provincial banks choose the coverage that exactly suffices to repay all the liability. Letting $\gamma = 1$ in (38) we find the optimal issuance

$$M = \left(\frac{q\alpha\xi}{(1+\upsilon)(1+\rho)}\right)^{\frac{1}{1-\alpha}} := M(\upsilon). \tag{40}$$

To find C, observe that in the symmetric equilibrium $M = \widetilde{M}$, putting which into $M(\Lambda) = \frac{1}{1+\rho}\left(\Lambda + \rho\beta\widetilde{M}\right)$,

$$\Lambda = (1 + (1-\beta)\rho) M. \tag{41}$$

As the insurance coverage $C = \gamma\Lambda = \Lambda$, we have

$$C = (1 + (1-\beta)\rho)\left(\frac{q\alpha\xi}{(1+\upsilon)(1+\rho)}\right)^{\frac{1}{1-\alpha}} := C(\upsilon).$$

Now we turn to the case in which $\upsilon < 0$. In this case, buying insurance benefits provincial banks in itself, besides offering coverage in the unfavorable contingency. Therefore, they want to buy as much insurance as possible, that is, as long as they can afford the insurance premium in the contingency of their loans performing. That is, constraint (26) is binding:

$$M(1+r) = (1+\mu\gamma)\Lambda,$$

which, together with $r = M^{-(1-\alpha)}\xi - 1$ by (37) and $\Lambda = (1 + (1-\beta)\rho) M$ by (41), implies that

$$\gamma = \frac{M^{-(1-\alpha)}\xi - (1 + (1-\beta)\rho)}{\mu(1 + (1-\beta)\rho)}. \tag{42}$$

Binding of (26) also suggests that constraint $\gamma \geq 1$ is not binding, as will be strictly confirmed later. Let λ be the Lagrangian multiplier of constraint (26) and substitute $M^{-(1-\alpha)}\xi - 1$ for r

in provincial banks' problem given in (30). Then the Lagrangian of the problem is $\mathcal{L}(\Lambda, \gamma) = q\left(\xi M^{\alpha} - (1+\mu\gamma)\Lambda\right) + (1-q)\Lambda \times (\gamma - 1) + \lambda\left(\xi M^{\alpha} - (1+\mu\gamma)\Lambda\right) = (q+\lambda)\left[\xi M^{\alpha} - (1+\mu\gamma)\Lambda\right] + (1-q)\Lambda \times (\gamma - 1)$, with $M = \frac{1}{1+\rho}\left(\Lambda + \frac{\widetilde{M}}{2}\rho\right)$. Hence,

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \Lambda} &= \frac{q+\lambda}{1+\rho}\xi\alpha M^{\alpha-1} - (1+\lambda) - (\upsilon + \lambda\mu)\gamma \\ \frac{\partial \mathcal{L}}{\partial \gamma} &= -(\upsilon + \lambda\mu)\Lambda. \end{aligned}$$

Then the first order conditions (FOCs) are thus:

$$\begin{aligned} \frac{q+\lambda}{1+\rho}\xi\alpha M^{\alpha-1} &= (1+\lambda) + (\upsilon + \lambda\mu)\gamma \\ \upsilon + \lambda\mu &= 0. \end{aligned}$$

The second equation implies that $\lambda = -\frac{\upsilon}{\mu}$. It follows that $1 + \lambda = 1 - \frac{\upsilon}{\mu} = 1 - \frac{q\mu - (1-q)}{\mu} = (1-q) \times \frac{1+\mu}{\mu}$ and $q + \lambda = q - \frac{q\mu-(1-q)}{\mu} = \frac{1-q}{\mu}$. Substitute these into the first FOC and we find $\frac{1-q}{(1+\rho)\mu}\xi\alpha M^{\alpha-1} = (1-q) \times \frac{1+\mu}{\mu}$, which leads to

$$\begin{aligned} M(\upsilon) &= \left(\frac{\xi\alpha}{(1+\rho)(1+\mu)}\right)^{\frac{1}{1-\alpha}} \\ |_{1+\mu=\frac{1+\upsilon}{q}} &= \left(\frac{q\xi\alpha}{(1+\rho)(1+\upsilon)}\right)^{\frac{1}{1-\alpha}}, \end{aligned} \tag{43}$$

the same as (40), the value of M in the case of $\upsilon > 0$.

To find the insurance coverage, note that from (42) and (43), $\gamma = \frac{(1+\rho)(1+\mu) - \alpha(1+(1-\beta)\rho)}{\alpha\mu(1+(1-\beta)\rho)}$, which, as $\mu = \frac{\upsilon+1-q}{q}$, leads to

$$\begin{aligned} \gamma &= \frac{(1+\rho)(1+\upsilon) - \alpha q(1+(1-\beta)\rho)}{\alpha(1-q+\upsilon)(1+(1-\beta)\rho)} \\ &= 1 + \frac{(1+\upsilon)\left[1-\alpha+\rho(1-\alpha+\alpha\beta)\right]}{\alpha(1-q+\upsilon)(1+(1-\beta)\rho)} \end{aligned}$$

As $\Lambda = (1+(1-\beta)\rho)M$ by (41) and $C = \gamma\Lambda$, we find $C(\upsilon) = \gamma(1+(1-\beta)\rho) = \frac{(1+\rho)(1+\upsilon)-\alpha q(1+(1-\beta)\rho)}{\alpha(1-q+\upsilon)}M(\upsilon)$. Here we also have confirmed that $\gamma(\upsilon) > 1$ all the time and hence constraint $\gamma \geq 1$ is not binding, as was said. Q.E.D.

Proof of Proposition 3:

Proof. It suffices to prove that the optimal $\upsilon^* = q\mu^* - (1-q)$ satisfies $\partial\upsilon^*/\partial\rho < 0$. As υ^* satisfies the first order condition $\Pi'_{\upsilon} = 0$, by the Implicit Function Theorem, $\partial\upsilon^*/\partial\rho =$

$-\Pi^{''}_{\upsilon\rho}/\Pi^{''}_{\upsilon\upsilon}$. The second order condition of the maximization problem commands that at $\upsilon = \upsilon^*$, $\Pi^{''}_{\upsilon\upsilon} < 0$. Hence, $\partial \upsilon^*/\partial \rho$ has the same sign as $\Pi^{''}_{\upsilon\rho} = (1-\beta)\,\rho M'(\upsilon) < 0$ because $M(\upsilon) = \left(\frac{\xi\alpha}{(1+\rho)(1+\mu)}\right)^{\frac{1}{1-\alpha}}$ according to Lemma (3). Q.E.D. ■

Proof of Lemma 1:

We prove the lemma for two cases, depending on the sign of υ. First, consider the case in which $\upsilon \geq 0$, namely, the capital bank obtains profit from providing insurance. In this case, by (27), $z_C = \frac{1}{1+\mu} - \frac{(1-\beta)\rho M(\mu)}{(1+\mu)C(\mu)} < \frac{1}{1+\mu} = \frac{q}{1+\upsilon} \leq q$ if $\upsilon \geq 0$.

Second, consider the case of $\upsilon < 0$. By (27), $z_C = \frac{C(\mu)-(1-\beta)\rho M(\mu)}{(1+\mu)C(\mu)}$, which, as $C = \chi(\upsilon)M$ by Lemma 3, equals$\frac{\chi(\upsilon)-(1-\beta)\rho}{(1+\mu)\chi(\upsilon)}$. Therefore, $z_C < q \Leftrightarrow \frac{\chi(\upsilon)-(1-\beta)\rho}{(1+\mu)\chi(\upsilon)} < q|_{q(1+\mu)=(1+\upsilon)} \Leftrightarrow \chi(\upsilon) - (1-\beta)\,\rho < (1+\upsilon)\,\chi(\upsilon) \Leftrightarrow$

$$\chi(\upsilon) < \frac{(1-\beta)\,\rho}{-\upsilon}. \tag{44}$$

at $\upsilon = \upsilon^*$. To prove this inequality, we go to the first order condition (FOC) for the principal part of υ^*, that is, $\Pi'_\upsilon = 0$, which, with $\Pi(\upsilon,\rho) = [\upsilon\chi(\upsilon) + (1-\beta)\,\rho]\,M(\upsilon)$, is equivalent to $[\upsilon\chi'(\upsilon) + \chi(\upsilon)]\,M(\upsilon) + [\upsilon\chi(\upsilon) + (1-\beta)\,\rho]\,M'(\upsilon) = 0$. As $M(\upsilon) = \left(\frac{q\xi\alpha}{(1+\rho)(1+\upsilon)}\right)^{\frac{1}{1-\alpha}}$ by Lemma *3*, $M'(\upsilon) = \frac{-1}{(1-\alpha)(1+\upsilon)}M$. Therefore, the FOC is equivalent to $[\upsilon\chi'(\upsilon) + \chi(\upsilon)] + [\upsilon\chi(\upsilon) + (1-\beta)\,\rho]\,\frac{-1}{(1-\alpha)(1+\upsilon)} = 0 \Leftrightarrow$

$$\chi(\upsilon)\left(1 - \frac{\upsilon}{(1-\alpha)\,(1+\upsilon)}\right) - \frac{(1-\beta)\,\rho}{(1-\alpha)\,(1+\upsilon)} = -\upsilon\chi'(\upsilon). \tag{45}$$

By Lemma *3*, $\chi(\upsilon) = \frac{(1+\rho)(1+\mu)-\alpha(1+(1-\beta)\rho)}{\alpha\mu} = \frac{1}{\mu}\left[\frac{(1+\rho)}{\alpha} - \left(1+\frac{\rho}{2}\right)\right] + \frac{1+\rho}{\alpha}$ with $\mu = \frac{\upsilon+1-q}{q}$. As $\frac{(1+\rho)}{\alpha} - \left(1+\frac{\rho}{2}\right) > 0$, we have $\chi'(\upsilon) < 0$. Therefore, if $\upsilon < 0$, then the right hand side of (45) is negative. It follows that $\chi(\upsilon)\left(1 - \frac{\upsilon}{(1-\alpha)(1+\upsilon)}\right) - \frac{(1-\beta)\rho}{(1-\alpha)(1+\upsilon)} < 0 \Leftrightarrow \chi(\upsilon)\frac{1-\alpha-\alpha\upsilon}{(1-\alpha)(1+\upsilon)} - \frac{(1-\beta)\rho}{(1-\alpha)(1+\upsilon)} < 0 \Leftrightarrow \chi(\upsilon) < \frac{(1-\beta)\rho}{1-\alpha-\alpha\upsilon}$, which leads to inequality (44) if $1-\alpha-\alpha\upsilon > -\upsilon > 0$. $-\upsilon > 0$ because we are considering the case of $\upsilon < 0$. To prove $1-\alpha-\alpha\upsilon > -\upsilon$ or equivalently $\upsilon > -1$, observe that however strongly the capital bank wants to encourage provincial banks to increase lending by reducing the insurance premium μ, it would never chooses $\mu \leq 0$ because otherwise provincial banks would choose $\gamma = \infty$, namely demand an infinite amount of insurance thereby obtaining such an amount of profit. With $\upsilon = q\mu - (1-q)$, we have

$$\upsilon > -(1-q) > -1.$$

Q.E.D.

Proof of Lemma 5:

By the implicit function theorem,

$$\Gamma'(s) = \frac{G'(s)}{G'(1-f)} \times \frac{1-\widetilde{\delta}}{\widetilde{\delta}}.$$

Therefore $\Gamma'(s) > 0$. We saw that by the Central Limit Theorem, $G(z) \approx \Phi\left(\frac{(z-q)\sqrt{N}}{\sqrt{q(1-q)}}\right)$. Hence, $G'(z) \approx \frac{1}{\sqrt{2\pi}} e^{-\frac{(z-q)^2 N}{2q(1-q)}}$. It follows that

$$\frac{G'(s)}{G'(1-f)} \approx e^{-\frac{N}{2q(1-q)} \times \left[(s-q)^2 - (1-f-q)^2\right]}.$$

To prove the second part of the lemma, it suffices to show that at $\left(s=0, f=\underline{f}\right)$, $(s-q)^2 - (1-f-q)^2 > 0 \Leftrightarrow q^2 > \left(1-\underline{f}-q\right)^2$, which, if $1-\underline{f} < q$, is equivalent to $q > q - \left(1-\underline{f}\right)$ and obviously holds true, and if $1-\underline{f} \geq q$ is equivalent to $q > 1-\underline{f}-q \Leftrightarrow \underline{f} > 1-2q$, which obviously holds true as $q > \frac{1}{2}$ has been assumed.

To prove the last part of the lemma, note that at $s = z_P$, $1-f = z_P$ as well. Thus $\Gamma'(s) = \frac{G'(s)}{G'(1-f)} \times \frac{1-\widetilde{\delta}}{\widetilde{\delta}} = \Gamma'(s) = \frac{1-\widetilde{\delta}}{\widetilde{\delta}}$. Q.E.D.

Proof of Lemma 6:

The lemma is equivalent to $1-\Gamma(s) < 1-(1-q)/q \times s \Leftrightarrow G(1-\Gamma(s)) < G(1-(1-q)/q \times s)$. By (36) $f = \Gamma(s)$ is defined by $G(z_P) - G(s) = [G(1-f) - G(s)]\widetilde{\delta} \Leftrightarrow G(1-\Gamma(s)) = \frac{G(z_P) - \left(1-\widetilde{\delta}\right)G(s)}{\widetilde{\delta}}$. It follows that the lemma is equivalent to $\frac{G(z_P) - \left(1-\widetilde{\delta}\right)G(s)}{\widetilde{\delta}} < G\left(1-\frac{1-q}{q}s\right) \Leftrightarrow$

$$G(z_P) < \widetilde{\delta} G\left(1-\frac{1-q}{q}s\right) + \left(1-\widetilde{\delta}\right)G(s) := y(s) \tag{46}$$

for $s \leq z_P$. Note at $s=0$, this inequality is equivalent to $G(z_P) < \widetilde{\delta}$, which holds true because $\lim_{N\to\infty} G(z_P) = 0$. Hence, inequality (46) follows from $y'(s) > 0$, which is proven as follows. $y'(s) = -\frac{1-q}{q}\widetilde{\delta}G'\left(1-\frac{1-q}{q}s\right) + \left(1-\widetilde{\delta}\right)G'(s)$. Observe that (i) by the Central Limit Theorem, $G'(z) \approx \frac{1}{\sqrt{2\pi}} e^{-\frac{N}{2q(1-q)} \times (z-q)^2}$ and thus decreases with $|z-q|$. Furthermore, because $s \leq z_P < q$, we have $1-\frac{1-q}{q}s > q$. It follows that (ii) $|1-\frac{1-q}{q}s - q| = 1-\frac{1-q}{q}s - q$ and $|s-q| = q-s$. Lastly, (iii) $1-\frac{1-q}{q}s-q < q-s$ because that is equivalent to $s\frac{2q-1}{q} < 2q-1|_{2q-1>0} \Leftrightarrow s < q$, which holds true. With these preparations, we come to prove $y'(s) > 0$. The last two claims together imply $|1-\frac{1-q}{q}s-q| > |s-q|$, which together with claim (i) implies $G'\left(1-\frac{1-q}{q}s\right) < G'(s)$. It follows that $y'(s) = -\frac{1-q}{q}\widetilde{\delta}G'\left(1-\frac{1-q}{q}s\right) + \left(1-\widetilde{\delta}\right)G'(s) > \left[-\frac{1-q}{q}\widetilde{\delta} + \left(1-\widetilde{\delta}\right)\right]G'(s) = \left[1-\frac{1}{q}\widetilde{\delta}\right]G'(s) > 0$, which holds true because we have assumed $\delta < q(1-q)$ in (32) and hence $\widetilde{\delta} = \frac{\delta}{1-q} < q$. Q.E.D.

Proof of Proposition 5:

If $s(1) < z_P$, then $f(1) = 1 - s(1) > \Gamma(s(1))$ because $\Gamma(s) < 1 - s$ for $s < z_P$ by the discussion preceding Lemma 5. Therefore any path ends at $(f(1), s(1))$ enters zone Ω. If $s(1) \geq 1 - \underline{f}$, then for any z, $f(z) \leq f(1) = 1 - s(1) \leq \underline{f} = \Gamma(0) \leq \Gamma(s(z))$, where the first inequality of the chain holds because a path $f'(z) \geq 0$ by definition and the last one holds because $\Gamma'(s) > 0$ by Lemma 5. Therefore, no paths $(f(z), s(z))$ enter Ω. Lastly, if $s(1) \in (z_P, 1 - \underline{f})$, we can construct a path that enters Ω and a path that does does not. For the former, let $(f(z), s(z)) = \left\{ \begin{array}{c} (z, 0) \text{ for } z \leq 1 - s(1) \\ (1 - s(1), z - 1 + s(1)) \text{ for } z \geq 1 - s(1) \end{array} \right\}$, that is, all the negative shocks occur in the front part of the stage; this path enters Ω because at $z = 1 - s(1)$, $f(z) = 1 - s(1)|_{s(1)<1-\underline{f}} > \underline{f} = \Gamma(s(z))$. For the latter, let $(f(z), s(z)) = \left\{ \begin{array}{c} (0, z) \text{ for } z \leq s(1) \\ (z - s(1), s(1)) \text{ for } z \geq s(1) \end{array} \right\}$, that is, all the positive shocks occur in the front part of the stage; this path does not enter Ω because for any z such that $s(z) \leq z_P < s(1)$, $f(z) = 0 < \Gamma(s(z))$. Q.E.D.

Proof of Proposition 6:

Consider $g(\tau) := \Gamma(\tau) - \frac{1-q}{q}\tau$ over $\tau \in [0, z_P]$. By Lemma 6, $g(\tau) > 0$. By Lemma 5, $g'(0) = \Gamma(0) - \frac{1-q}{q} < 0$ and $g'(z_P) = \frac{1-\widetilde{\delta}}{\widetilde{\delta}} - \frac{1-q}{q} > 0$ because $\widetilde{\delta} = \frac{\delta}{1-q} < q$ by the assumption in (32). Define $f_a := \min_{\tau \in [0, z_P]} g(\tau)$. Then $f_a > 0$ and $f_a < g(0) = \underline{f}$. We prove that for any $f \in [f_a, \underline{f}]$, $(f, 0) \in \Phi$. For such a f obviously $f \leq \Gamma(0) = \underline{f}$. Thus we only need to prove that $f + \frac{1-q}{q}\tau > \Gamma(\tau)$ for some $\tau > 0$, which is equivalent to $f > g(\tau)$ for some $\tau > 0$, which holds true because $f \geq f_a$ and thus $f \geq \min_{\tau \in [0, z_P]} g(\tau)$. Q.E.D.

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