Diffusing Coordination Risk

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Abstract

Agents face strategic uncertainty in a coordination problem that is akin to debt rollover or currency attacks. We model this as a global game of regime change. A principal wants her preferred regime (PPR) to succeed. She faces the coordination risk that a viable PPR may fail due to the strategic uncertainty. The principal diffuses this coordination risk by making a finite partition of the mass of agents. She abandons her preferred regime if it is no longer viable. We show that with a sufficiently diffused policy, the risk that agents may attack the PPR unravels from the end.

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Introduction

Coordination games give rise to multiple equilibria. The strategic uncertainty that agents face concerning the actions and beliefs of others may lead to an undesirable outcome. For example, pessimistic investors who are wary of the non-participation of other potential investors may decide

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to walk away from a profitable investment opportunity. Countries that try to attract foreign investments often do so to no avail. Similarly, borrowers may fail to persuade their creditors to roll over their debt claims despite sound fundamentals. In this paper, we take the global game approach to model this coordination risk. Suppose that a principal, similar to a Pigouvian planner, tries to achieve the desired outcome that the market fails to achieve. Sakovics and Steiner (2012) and Cong et al. (2016) find the optimal investment subsidies that, at a given cost, maximize the likelihood of successful coordination.[1] Unlike the planner in the above mentioned papers, we consider a principal who cannot offer monetary incentives. However, the principal can provide some additional information to convince the agents to follow her recommendations, as in Bergemann and Morris (2013), Inostroza and Pavan (2017) or Goldstein and Huang (2016). In this paper, we propose a simple policy through which a principal can eliminate all the coordination risk.

Imagine that there are two regimes, a mass of agents and a principal. Each agent decides which regime to attack. In the end, only one of these two regimes succeeds. The principal strictly prefers one regime over the other. We refer to this regime as the principal's preferred regime (**PPR**). The principal wants the PPR to succeed in the end. However, it will succeed if and only if the PPR has sufficient fundamental strength to withstand the aggregate attack. For example, a borrower survives in the end if her liquidity holding (the fundamental strength) is greater than the total withdrawal from her creditors (aggregate attack). Although the agents prefer not to attack the regime that succeeds, they are uncertain as to which regime will succeed.

We assume that the agents are uncertain about the underlying fundamental strength and receive noisy private information about it. This game is commonly referred to as global game of regime change.[2] Global game theory predicts a unique threshold strength, such that the PPR succeeds if and only if it has strength beyond this threshold. We say that a PPR is not "viable" if it fails even when no agent attacks it. Thus, a viable PPR may fail because of strategic uncertainty if it is not sufficiently strong. We call this probability of failure - the coordination risk.

The principal adopts a simple policy. We call this policy, diffusing coordination risk. The policy

[1] While Sakovics and Steiner (2012) consider heterogenous agents and show that subsidizing the more reluctant agents matters more, Cong et al. (2016) show that in a dynamic coordination problem if liquidity injection is equally costly across periods, then early injection is more helpful.

[2] For examples of a global game of regime change, see Morris and Shin (1998) and Angeletos et al. (2007) for the currency crisis, Goldstein and Pauzner (2005) for self-fulfilling bank runs, Vives (2014) for financial fragility, Edmond (2013) for riots and political change and Konrad and Stolper (2016) for fight against tax havens. For more recent developments see Szkup and Trevino (2015).

makes a finite partition of the mass of agents. Agents within the same group make their decisions simultaneously while different groups move chronologically. The principal commits to the rule that she will abandon her preferred regime if it is no longer viable. Thus, when the game reaches group t, the principal endogenously and publicly transmits the information whether the PPR continues to be viable. We refer to this as public news of continued viability - **PNV**.

In the spirit of the Bayesian persuasion literature after Kamenica and Gentzkow (2011) and Ely (2017), we can think of our principal as an information designer. The principal does not know the private information that any agent receives. However, she has an information advantage over the agents in the sense that she can verify the viability of her preferred regime. The principal decides to whom she discloses this information. This policy can be interpreted as a simple recommendation "to attack" the PPR when it is abandoned and "not to attack" when the PPR is not abandoned. If the principal abandons the PPR, then the agents learn that the PPR is no longer viable regardless of their actions. Hence, following the principal's recommendation and attacking the PPR is their dominant strategy. On the other hand, if the principal does not abandon the PPR, then the agents learn that it has survived the attacks so far. Is this information sufficiently strong to convince the agents to follow the principal's recommendation and not attack the PPR? In this paper, we show that if all the groups are sufficiently small in size, there is a **unique** equilibrium in which the agents indeed ignore their private information and follow the principal's recommendation. In other words, sufficient diffusion eliminates the coordination risk.

We share a close conceptual connection with Bayesian persuasion and information design for heterogeneously informed multiple receivers as defined by Bergemann and Morris (2016).[3] However, the basic question is different. The information design literature considers the general scope of information structure and asks the question: Is it possible to design the information structure to achieve a certain outcome in equilibrium? In reality, we do not expect that a principal can have full control over designing the information structure. So, we look into a specific type of information structure that is less demanding in the sense that it does not require additional ex-ante **commitment power** of the principal and ask the question: Can this simple information structure achieve a desirable outcome in any possible equilibria? Recall that the principal wants her preferred

[3]Mathevet et al. (2017) and Taneva (2016) consider a more general persuasion setting for multiple receivers. Ely (2017) considers information disclosure in a dynamic setting. See Alonso and Câmara (2016a), Alonso and Câmara (2016b), Bardhi and Guo (2016) for persuasion in the context of voting.

regime to succeed in the end. If she learns that the PPR is not viable - i.e., it will fail regardless of the actions of the agents in the subsequent groups, then she has no reason to convince the agents not to attack it.

Coming back to the debt rollover problem, this policy is equivalent to an asynchronous debt structure. Only a fraction of short term debts mature at a time and if most of them are not rolled over, then the borrower may become illiquid and fail. The principal wants the borrower to succeed but if she learns that the borrower is bound to fail, regardless of the creditors' decisions, then she abandons defending the borrower. We show that a sufficiently asynchronous debt structure can make the borrower immune to panic-based runs.

When agents move simultaneously, there is no information transmission. Agents completely rely on their private information to infer what information others have and what they are likely to do. The partition alone does not change this. However, by diffusing the coordination risk, the principal transmits the public information of continued viability. Angeletos et al. (2007) study the same type of public information in the context of currency attacks. Similar to our model, Goldstein and Huang (2016) and Inostroza and Pavan (2017) also consider a principal who uses such information as a tool to convince the agents not to attack the PPR. They consider a principal who recommends "Not to Attack" the PPR only when the PPR has strength beyond a certain threshold. In the context of banking, this can be interpreted as a "stress test". The authors show that if the principal can "commit" to such a rule with a sufficiently high threshold, then agents will follow the recommendation. Inostroza and Pavan (2017) find the condition under which such public disclosure is optimal. Observe that below this policy threshold, the PPR still fails. Thus, this policy cannot achieve the first best outcome. Also, the ex-ante commitment is crucial for this policy. If the principal knows that agents will not attack the PPR if she recommends them to do so, then she has an incentive to lie and recommend the same, even when the PPR does not have strength beyond the announced threshold. As mentioned before, our principal does not face such a commitment issue.

Our principal only discloses that the PPR is viable as opposed to disclosing that the PPR is sufficiently strong as in Goldstein and Huang (2016). Thus, the evidence in itself is not convincing enough for the agents not to attack the PPR. However, if only a small fraction of agents are facing this strategic uncertainty, then we show that this information can be sufficiently strong to convince the agents not to attack the PPR. Imagine that in a static game, the actual strength is θ and there are α mass of agents playing this game. The PPR will survive if θ is large enough to sustain attack from α mass of agents. Thus, the effective strength is θ/α. Suppose that the agents were receiving some noisy private information $s = \theta + \sigma\epsilon$, where ϵ is a random noise and σ is a scale parameter. Then we can think of s/α as the private information about θ/α. Note that

$$\frac{s}{\alpha} = \frac{\theta}{\alpha} + \frac{\sigma}{\alpha}\epsilon.$$

Thus, the scale parameter σ/α is inversely proportional to α - i.e., the private information becomes a very noisy predictor of the effective strength as α becomes very small. We show that as the private information becomes sufficiently noisy, agents indeed ignore their private information and follow the principal's recommendation.

When the principal sufficiently diffuses the coordination risk, there are several groups with a sufficiently small mass of agents in each group. After learning that the PPR has survived so far, agents in these groups are not only concerned regarding the beliefs and actions of the other agents in their group (current risk) but also, regarding the actions and beliefs of the agents in the subsequent groups (future risk). Let us first consider the agents in group 1. Suppose that they believe that if the PPR survives the current attacks, it will survive all the future attacks - i.e., there is no future risk. Then they are essentially facing a static regime change problem. Therefore, as argued before, if the group size is sufficiently small, the agents will ignore their private information and follow the principal's recommendation. Next, we argue that the effect of PNV is stronger as the PPR survives attacks from a larger mass of agents. Thus, given no future risk and the same group size, if agents in group $t = 1$ ignore their private information and follow the principal's recommendation, then so will the agents in the subsequent groups.

The above argument shows that the principal can convince the agents not to attack the PPR, provided that she can convince them that there is no future risk. In other words, the principal needs to convince the agents that the agents in the subsequent groups will also follow her recommendation. Since the agents in the last group do not face any future risk, they will surely follow the recommendation. Therefore, the agents in the penultimate group also face no future risk and follow the recommendation and so on. Thus, the coordination risk unravels from the end.

Since we are considering a private information world, this unraveling is not a standard backward induction argument. Formally, we show that there is a uniform critical mass α^* such that when the mass of agents α_t in any group t is less than α^*, if the coordination risk unravels until $(t+1)$ (no agent will attack the PPR if it survives the current attack from group t), then it also unravels until t. Given no future risk for the last group, we establish the unraveling of the coordination risk from the end.

In the main text, we consider a standard regime change game and abstract away from some particular aspects that come with specific examples. In the selected application section, we show how our proposed policy works for many prevalent applications of the regime change game. Capital control policies like setting an annual limit for capital outflow, could work to deter the currency attacks and to keep the foreign exchange rate less volatile. Sufficiently asynchronous debt structures could help the borrower to avoid panic-driven debt runs. In reality, we can see that the suggested policies have been applied to the foreign exchange and debt markets. For example, the central bank in China has imposed the ceiling for converting the domestic currency to foreign assets for a long time in order to regulate the foreign exchange rate (see Neely (2017)). Corporate bond issuers adopt the suggested policies through the diversification of debt rollovers across maturity dates (Choi et al. (2016)). Note that such policies can make the borrower (or the country) immune to panic-based runs (currency attacks). However, adopting such a policy only in times of distress could have a severely adverse impact on agents' beliefs about the solvency of the borrower (country's commitment to maintain the exchange rate). Ex-ante announcement of the suggested policy precludes this signaling problem.

Asynchronous move in coordination games is hardly a novel contribution. Gale (1995) shows that if agents can choose to delay their actions in a coordination game, they indeed do so in equilibrium. This endogeneity of timing is missing in our paper. While Gale considers finite agents, Dasgupta (2007) models a continuum of agents moving sequentially in two groups. Moreover, unlike Gale, the author assumes that agents do not perfectly observe the past actions and shows that the probability of successful coordination increases under sequential decisions. In this paper, the endogenous information transmission is through the public information of continued viability, which is in contrast to the learning from noisy private information about past attacks as in Dasgupta (2007). However, we show that our result is robust to introducing additional information similar to Dasgupta (2007). The novelty of this paper is in the main result that a sufficiently fine partition can eliminate all the strategic uncertainty. Lagunoff and Matsui (1997) and Dutta (2012) show similar results in asynchronous move coordination games but they consider a repeated game and there is complete and perfect information. They show that mis-coordination disappears within a finite number of periods and agents start coordinating on the Pareto dominant outcome thereafter. In contrast, in this paper it is not necessary that the principal prefers the Pareto dominant outcome.

As discussed before, we consider a private information world as in the global game literature following Carlsson and Van Damme (1993). This theory provides a unique and intuitive monotone equilibrium selection in coordination games. For a comprehensive discussion of Global Game, see Morris and Shin (2003) and Morris et al. (2016). We borrow from two strands of papers within this global game literature. The first strand follows Angeletos et al. (2007) and explores the scope of public news of survival as a tool to persuade agents. Huang (2017) extends Angeletos et al. (2007) by considering a principal with a long run reputation concern. In Cong et al. (2016), the principal can influence such survival news by early liquidity injection. In this literature, agents may get multiple opportunities to attack but they do not have any incentive to coordinate with future actions which are influenced by their own actions. The second strand, such as Dasgupta et al. (2012) and Mathevet and Steiner (2013), on the other hand, focuses on the incentive to coordinate with future actions. However, neither of these two strands of literature alone explains how diffusing coordination risk persuades all the agents to ignore their private information.[4]

1 A Simple Model of Regime Change

1.1 Static Benchmark

Model Setup There are two regimes, indexed by $\mathcal{R} \in \{0, 1\}$, a continuum of agents, indexed by $i \in [0, 1]$ and a principal. Agents simultaneously decide which regime to attack. The action space for any agent i is $A_i = \{0, 1\}$. If he takes an action $a_i = \mathcal{R} \in \{0, 1\}$, then we say that the agent supports regime $\mathcal{R}$ or attacks the other regime. The principal prefers one regime over the other. Without loss of generality, we assume that regime 1 is the principal's preferred regime (PPR). The principal gets a payoff of 1 if the PPR succeeds and 0 otherwise. Therefore, the principal wants the PPR to succeed in the end. Let θ be the underlying fundamental strength of the PPR and

[4]The result has a flavor similar to the information cascading result of Banerjee (1992), Bikhchandani et al. (1992). The authors show that it is possible that after a few agents take the same action, others start following the herd, ignoring their own private information. Unlike the information cascading literature, agents are facing a coordination problem that involves static and dynamic strategic uncertainty. Under a sufficiently diffused policy, we show that herding is not just a possibility rather it will inevitably start from the beginning.

$w = \int_i \mathbb{1}(a_i = 0)di = \int_i (1 - a_i)di$ be the aggregate attack or the withdrawal from supporting the PPR. The PPR succeeds in the end if and only if its fundamental strength θ is strong enough to withstand the aggregate attack against it - i.e., $\theta \geq w$.

Agents are assumed to be ex-ante identical and risk-neutral. If the agent supports regime $\mathcal{R}$ and it succeeds, then he gets a payoff of $b_{\mathcal{R}}$. Whereas if he had attacked the regime that succeeds, then he would have received $c_{\mathcal{R}}$. We assume $b_{\mathcal{R}} > c_{\mathcal{R}}$ for any $\mathcal{R} \in \{0, 1\}$. If regime $\mathcal{R}$ succeeds, then agent i would be better off if he had not attacked it. This feature is usually referred to as strategic complementarity. Given the fundamental strength (θ) and the aggregate attack against the PPR (w), let $u(a_i, w, \theta)$ be the payoff[5] for agent i if he takes action a_i, where

$$u(1, w, \theta) = \begin{cases} b_1 & \text{if } \theta \geq w \\ c_0 & \text{if } \theta < w \end{cases}, \qquad u(0, w, \theta) = \begin{cases} c_1 & \text{if } \theta \geq w \\ b_0 & \text{if } \theta < w. \end{cases} \tag{1}$$

Agent i receives noisy private information about θ, denoted by

$$s_i = \theta + \sigma\epsilon_i, \tag{2}$$

where the error terms ϵ_i is conditionally independent[6] and identically distributed with zero mean. Let $F : [-1/2, 1/2] \to [0, 1]$ be the distribution and f be the density of the error. We assume that F is continuously differentiable and log-concave. This log-concavity implies that for any $a > 0$, $\frac{\partial}{\partial x}\left(F(x - a)/F(x)\right) > 0$.[7] σ scales the random noise ϵ_i. Nature selects the fundamental strength θ from the interval $[\underline{\theta}, \bar{\theta}]$ with uniform probability. We assume that $\underline{\theta} \leq -\sigma$ and $\bar{\theta} \geq 1 + \sigma$.[8] For any θ, the probability of receiving private signal s is distributed via $F\left((s - \theta)/\sigma\right)$. Given the uniform prior, for any $s \in [-\sigma/2, 1 + \sigma/2]$, θ is distributed via $1 - F\left((s - \theta)/\sigma\right)$. We call $\tau := 1/\sigma^2$ the precision of the private signal.

[5]We prove our results for a more general payoff structure. See the proof of Proposition 5 in the appendix. For simplicity, we use this payoff structure in the main paper.

[6]See Judd (1985) for the existence of a continuum of independent random variables.

[7]Inostroza and Pavan (2017) also make a similar assumption regarding the noise distribution. Most of the distributions we use in practice satisfy log-concavity, e.g., uniform, normal or exponential distribution. The exceptions are student's t, and the log-normal distributions.

[8]This assumption guarantees that the common prior is uninformative when agents have private information. If $s < -\sigma/2$ (or $s > 1 + \sigma/2$), agents know that $\theta < 0$ (or $\theta > 1$). These capture the dominance regions.

Equilibrium Analysis This is the standard static global game problem and readers familiar with this literature can skip this section. Agents receiving higher signals believe that the PPR is stronger against attacks. They also believe that other players believe that the PPR is stronger against attacks and so on (higher order beliefs).

If an agent believes that the PPR will succeed with probability greater than p, he will not attack it, where

$$p := \frac{1}{1 + \frac{b_1 - c_1}{b_0 - c_0}}.$$

We now characterize the monotone equilibrium where an agent i does not attack the PPR if and only if $s_i \geq s^*$ for some threshold s^*. In equilibrium, the higher the fundamental θ, the larger the share of agents who receive signals above s^* and do not attack the PPR. Thus, there exists θ^* such that the PPR succeeds if and only if $\theta \geq \theta^*$. The monotone equilibrium can be summarized by (θ^*, s^*). Following Morris and Shin (2003), this monotone equilibrium is indeed, the unique equilibrium.

Proposition 1 *There is a unique Bayes Nash equilibrium in which an agent plays $a_i = 1$ if and only if he receives a signal $s_i \geq s^*$. The PPR succeeds if and only if $\theta \geq \theta^*$, where $\theta^* = p$ and $s^* = \theta^* + F^{-1}(\theta^*)/\sqrt{\tau}$.*

If $\theta < 0$, then the PPR will fail regardless of the agents' action. We call such a PPR not **viable**. Due to strategic uncertainty, the PPR may fail even when it is viable. Let $P(0 \leq \theta < \theta^*)$ be this probability of failure. The above proposition uniquely identifies this probability, which we call the **coordination risk**. The principal wants to minimize θ^*, which is a sufficient statistic for the coordination risk. The first best outcome for the principal is $\theta^* = 0$.

1.2 Diffusing Coordination Risk

Imagine that the principal publicly announces the following policy: (1) Each agent will be randomly assigned to one of the T groups. Agents who are assigned to group $t = 1, 2, \ldots, T$, will take actions only once at time t,[9] and their actions are irreversible and (2) She will abandon her preferred regime if it is no longer viable. We refer to this policy as diffusing coordination risk (or in short the diffused

[9] The chronology is introduced to achieve sequentiality in the static game. Note that there is no discounting of payoffs.

policy) and denote it by $(T, (\alpha))$, where $T \in \mathbb{N}$ and $(\alpha) \equiv (\alpha_1, \alpha_2, \ldots, \alpha_T) \in$ the $T-1$ simplex $\Delta^{T-1} \equiv \{(\alpha_1, \alpha_2, \ldots, \alpha_T) \in \mathbb{R}^T | \ \alpha_t \geq 0 \text{ for all } t \in \{1, 2, \ldots, T\} \text{ and } \sum_{t=1}^{T} \alpha_t = 1\}$.

Let $a_{it} \in \{0, 1\}$ be the action taken by agent i in group t and w_t be the proportion of agents in group t who attack the PPR - i.e., $w_t = (1/\alpha_t) \int (1 - a_{it}) di$. The diffused policy restricts the magnitude of attack that can occur within one period - i.e., $\alpha_t w_t \leq \alpha_t$ - but does not change the maximum possible aggregate attack - i.e., $\sum_{t=1}^{T} \alpha_t = 1$. The preferred regime succeeds if and only if it withstands all attacks against it. The payoff under a diffused policy is

$$u(1, w, \theta) = \begin{cases} b_1 & \text{if } \theta \geq \sum_{t=1}^{T} \alpha_t w_t \\ c_0 & \text{if } \theta < \sum_{t=1}^{T} \alpha_t w_t \end{cases}, \qquad u(0, w, \theta) = \begin{cases} c_1 & \text{if } \theta \geq \sum_{t=1}^{T} \alpha_t w_t \\ b_0 & \text{if } \theta < \sum_{t=1}^{T} \alpha_t w_t. \end{cases} \tag{3}$$

The payoff in Equation (3) is exactly as given in Equation (1), where $w \equiv \sum_{t=1}^{T} \alpha_t w_t$.[10]

As in the benchmark model, agents share an uninformative prior and receive some noisy private information about the fundamental strength θ. In addition, this policy endogenously transmits the public information that the PPR continues to be viable. Let θ_t be the residual fundamental strength at time t. Then,

$$\theta_t := \theta_{t-1} - \alpha_{t-1} w_{t-1} = \theta_1 - \sum_{u=1}^{t-1} \alpha_u w_u \text{ for } t = 2, 3, \ldots, T \ \text{ and } \ \theta_1 = \theta. \tag{4}$$

If the PPR fails to withstand the attacks until $(t-1)$ - i.e., $\sum_{u=1}^{t-1} \alpha_u w_u > \theta_1$, then the residual strength $\theta_t < 0$. Therefore, the PPR will fail regardless of the actions of the agents in the subsequent groups. According to the proposed policy, the principal abandons such non-viable PPR. If the principal abandons the PPR, then the agents learn that the PPR cannot succeed. Thus, their dominant strategy is to attack the PPR. On the other hand, if she does not abandon the PPR, the agents publicly learn that the PPR is still viable - i.e., $\theta_t \geq 0$. We refer to this information as the **public news of continued viability** (PNV).[11]

[10]The payoff is realized at the end, depending on which regime succeeds. In many practical examples, payoff from a certain action does not depend on the final outcome. For example, in the case of a debt run game, a creditor may get his money back immediately after his withdrawal, even if the borrower fails in the future. We will address this issue in Section 3.

[11]Although we are assuming here that agents do not receive any additional information, in Section 4, we show that our result is robust, accounting for agents receiving additional private information regarding previous withdrawals.

Dynamic Information Design The proposed policy, "diffusing coordination risk" essentially designs the "information structure" - i.e., specifies what information is revealed to whom. The principal wants to convince a mass of agents not to attack a viable PPR. Imagine that a mass of agents is uniformly distributed over the time interval $[0, 1]$. Each agent receives private information and all agents make their decisions sequentially. They do not see others' actions and they cannot communicate. This is equivalent to the simultaneous move game we consider in Section 1.1. The policy $(T, (\alpha))$ induces the following information structure: The principal checks whether the PPR is still viable after each α_t units of elapsed time (after the initial check at time 0) and discloses the result publicly. Whether the PPR is still viable or not depends on the past actions of agents. Thus, the distribution over the signal realizations is history dependent. Since the actions are irreversible, the agents cannot change their decisions based on a public information that is disclosed at a later stage. If the PPR is still viable, the principal recommends not to attack it and if it is no longer viable, then she recommends to attack it.

Equilibrium We look into monotone equilibrium, which is a symmetric monotonic Perfect Bayesian Equilibrium. Any agent i in any group t plays a monotone strategy: $a_{it} = 1$ if and only if $s_i \geq s_t^*$. The PPR succeeds in the end iff $\theta = \theta_1 \geq \theta_1^*$. We define θ_t^* such that if the residual strength $\theta_t \geq \theta_t^*$, then the PPR withstands all attacks from t until T. We refer to this equilibrium as $(\theta_t^*, s_t^*)_{t=1}^T$. By definition, for any t, $\theta_t^* \geq \theta_{t+1}^*$.[12]

2 Equilibrium Characterization

Given a monotone equilibrium $(\theta_t^*, s_t^*)_{t=1}^T$, we define $\psi_t(\theta_t) := \theta_t - \alpha_t P(s < s_t^* | \theta_t)$ that maps the residual strength at t to the residual strength at $t+1$ and ψ^t that maps θ_1 to θ_{t+1} as

$$\theta_{t+1} := \psi_t \circ \psi_{t-1} \circ \ldots \psi_1(\theta_1) = \psi^t(\theta_1). \tag{5}$$

We use the convention, $\psi_0(\theta_1) = \theta_1$. In any monotone equilibrium, a higher initial strength implies that there will be less attack against the PPR and thus, there will be higher residual strength.

[12] By definition of θ_t^*, for any t, if $\theta_t \geq \theta_t^*$, then $\theta_{t+1} \geq \theta_{t+1}^*$. Since $\theta_t \geq \theta_{t+1}$, we can say that whenever $\theta_t \geq \theta_t^*$, $\theta_t \geq \theta_{t+1} \geq \theta_{t+1}^*$. Thus, $\theta_t^* \geq \theta_{t+1}^*$.

Consequently, these ψ_t, ψ^t are nondecreasing functions. We define the inverse function as

$$h_t(x) \equiv (\psi^{t-1})^{-1}(x),$$

which represents the fundamental strength at $t = 1$ that would have resulted in the residual fundamental strength x at time t. This is also a nondecreasing function. Note that these functions depend on the equilibrium specifications. The detailed information structure of a simple bifurcated diffusion is illustrated in Figure 1.

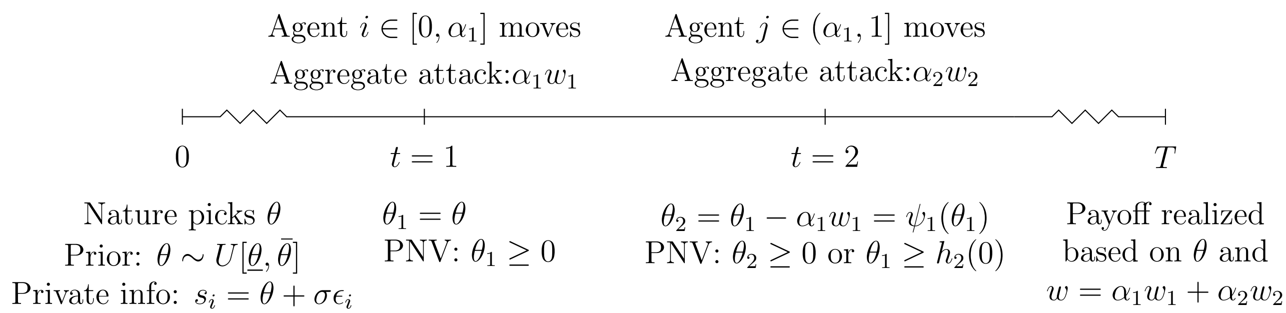

Figure 1: Timeline in a bifurcated diffusion

By definition of $h_t(.)$, if the initial fundamental strength $\theta_1 \geq h_t(0)$, then the PPR survives the attacks from the first $(t-1)$ groups. If the PPR survives attacks from a larger share of agents, then the initial strength must have been even higher. Thus, regardless of the equilibrium specification, $h_{t+1}(0) \geq h_t(0)$, for any t. If $\theta_1 \geq h_t(0)$, the principal does not abandon the PPR when the game reaches group t. However, the principal will abandon the PPR later, unless $\theta_1 \geq \theta_1^*$. By definition of θ_t^*, the PPR will succeed in the end only if the residual fundamental $\theta_t \geq \theta_t^*$. Mapping it back to the initial strength θ_1 using $h_t(.)$, this condition for success is equivalent to $\theta_1 \geq h_t(\theta_t^*)$. Thus, by definition of θ_1^*, for any $t = 1, 2, \ldots, T$,

$$h_t(\theta_t^*) = \theta_1^*.$$

For any $t = 1, 2, \ldots, T$, if $\theta_1 \in [h_t(0), h_{t+1}(0))$, then the PPR remains viable until the game reaches group t and then fails. If $\theta_1 = \theta_1^*$, then aggregate attack exactly exhausts all the fundamental strength - i.e., $\theta_{T+1} = 0$. Hence, $h_{T+1}(0) = \theta_1^*$. Figure 2 shows these different thresholds that capture when the PPR fails.

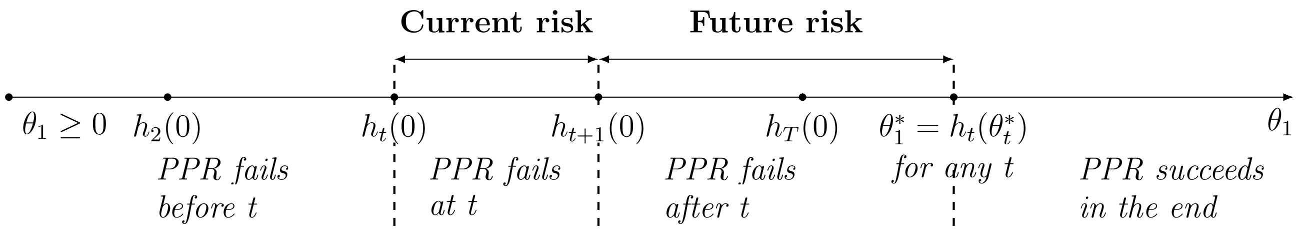

Figure 2: Current and Future Coordination Risk

After the game reaches group t, given the uniform prior, $\{h_t(\theta_t^*) - h_t(0)\}$ captures the coordination risk that the PPR which continues to be viable will not succeed in the end. This risk can be divided in two parts:

1. Current coordination risk: If $\theta_1 \in [h_t(0), h_{t+1}(0))$, then the PPR fails to survive the attacks from agents in group t and thus, the PPR will not be viable when the game reaches group $(t+1)$.

2. Future coordination risk: If $\theta_1 \in [h_{t+1}(0), h_t(\theta_t^*))$, then the PPR survives the current attack and thus, is viable when the game reaches group $(t+1)$ but will not survive future attacks.

If $\theta_t^* = 0$ or $\theta_1^* = h_t(0)$, then there is no coordination risk after the game reaches group t - i.e., if the PPR is viable when the game reaches group t, it will succeed in the end. Therefore, if $\theta_t^* = 0$, then $\theta_{t+1}^* = \ldots = \theta_T^* = 0$. The following proposition characterizes any monotone equilibrium.

Proposition 2 *When the coordination risk is diffused, there can be multiple equilibria. Any monotone equilibrium $(\theta_t^*, s_t^*)_{t=1}^T$ satisfies the following conditions:*

(a) For some $t' \in \{1, 2, \ldots, T+1\}$, $\theta_{t'}^* = \theta_{t'+1}^* = \ldots \theta_{T+1}^* = 0$.

(b) For $t \in \{1, 2, \ldots, t'-1\}$, θ_t^* satisfies the following recursive relation

$$\frac{\frac{\theta_t^* - \theta_{t+1}^*}{\alpha_t}}{F\left(F^{-1}\left(\frac{\theta_t^* - \theta_{t+1}^*}{\alpha_t}\right) + \sqrt{\tau}(h_t(\theta_t^*) - h_t(0))\right)} = p. \tag{6}$$

(c) For all $u \in \{1, 2, \ldots, T\}$,

$$s_u^* = \theta_1^* + \frac{1}{\sqrt{\tau}} F^{-1}\left(\frac{\theta_u^* - \theta_{u+1}^*}{\alpha_u}\right). \tag{7}$$

The above recursive relations do not necessarily have a unique fixed point. Thus, unlike the unique equilibrium in the benchmark model in Section 1.1, there are multiple equilibria. Such multiplicity arises even in a static problem due to the PNV. Consider a particular case: $t' = 1$ in point (a) of Proposition 2. Then $\theta_t^* = 0$ for all $t \in \{1, 2, \ldots, T\}$. For a viable PPR, if agents believe that no agent attacks the PPR, then it is rational for them not to attack the PPR. Recall that this is the first best the principal can achieve and it is always an equilibrium.

In general, given the fundamental thresholds $(\theta_u^*)_{u=1}^T$, Equation (7) represents the threshold signals s_u^* such that at $\theta_1 = \theta_1^*$, the proportion of agents in group u who receive $s_u < s_u^*$ and hence attack the PPR, is equal to $(\theta_u^* - \theta_{u+1}^*)/\alpha_u$ - i.e.,

$$w_u(\theta_1 = \theta_1^*) = F\left(\sqrt{\tau}(s_u^* - \theta_1^*)\right) = \left(\frac{\theta_u^* - \theta_{u+1}^*}{\alpha_u}\right).$$

Thus, when $\theta_1 = \theta_1^*$, the total attack against the PPR from time t to T is

$$\sum_{u=t}^{T} \alpha_u \left(\frac{\theta_u^* - \theta_{u+1}^*}{\alpha_u}\right) = \theta_t^*.$$

Hence, if $\theta_1 = \theta_1^*$, the PPR is just strong enough to withstand all attacks (from time 1 to T) and the residual fundamental strength at any t is just strong enough to withstand all future attacks (from time t to T) - i.e., $\theta_t = \theta_t^*$.

Imagine an agent in group t who receives the threshold signal s_t^*. He believes that the PPR will succeed with probability

$$P(\theta_1 \geq \theta_1^* | s_i = s_t^*, \theta_1 \geq h_t(0)) = \frac{P(\theta_1 \geq \theta_1^* | s_i = s_t^*)}{P(\theta_1 \geq h_t(0) | s_i = s_t^*)}.$$

The LHS of Equation (6) captures this belief. If $\theta_t^* > 0$, in equilibrium this threshold agent is indifferent between attacking and not attacking. Therefore, this belief equals p.

To solve for the sequence $(\theta_t^*)_{t=1}^T$ from the above recursive relation, we take a candidate solution θ_1^*. If $\theta_1^* = 0$, then $\theta_t^* = 0$ for all t. Otherwise, let us consider the case $\theta_1^* > 0$. Since $h_1(\theta) = \theta$, we can solve recursive relation (6) for $\theta_2^*(\theta_1^*)$. From Equation (7), we can solve for $s_1^*(\theta_1^*, \theta_2^*(\theta_1^*))$. Given s_1^*, we have $\psi_1(.|\theta_1^*)$ and hence $h_2(.|\theta_1^*)$. We can use recursive relation (6) and (7) again for θ_3^* and so on. Finally, if $\theta^*_{t+1} = 0$ for some $t \in \{1, 2, \ldots, T\}$, then we must have

$$\frac{\frac{\theta^*_t(\theta^*_1)}{\alpha_t}}{F\left(F^{-1}\left(\frac{\theta^*_t(\theta^*_1)}{\alpha_t}\right) + \sqrt{\tau}(h_t(\theta^*_t(\theta^*_1)|\theta^*_1) - h_t(0|\theta^*_1))\right)} = p.$$

2.1 Relation between Current Risk and Future Risk

Recall that $\{h_t(\theta^*_t) - h_{t+1}(0)\}$ captures the future risk for agents in group t. Since $h_{t+1}(\theta^*_{t+1}) = h_t(\theta^*_t)$, this future risk is also the aggregate risk that group $(t+1)$ faces, $\{h_{t+1}(\theta^*_{t+1}) - h_{t+1}(0)\}$. Thus, if $\theta^*_{t+1} = 0$, then agents in group t face no future risk. Then the entire risk for agents in group t comes from the agents within their own group.

The current risk, on the other hand, becomes zero if $h_t(0) = h_{t+1}(0)$. This implies that if the PPR is viable when the game reaches group t, it is also viable when the game reaches the next group. The next corollary shows that, if there is no current risk for group t, once the game reaches group t, it is not only guaranteed to remain viable when the game reaches the next group but also guaranteed to succeed in the end.

Corollary 1 *If agents in any group t face no current coordination risk, - i.e. $h_t(0) = h_{t+1}(0)$, then the future coordination risk is also 0. This implies, $\theta^*_{t+1} = \theta^*_t = 0$.*

There is no current risk at t only if the agents in group t ignore their private information and never attack a viable PPR. However, these agents are not only concerned with the current attacks but also the future attacks. Therefore, to persuade the agents in group t to refrain from attacking the PPR, it is necessary to eliminate the future risk. In the next subsection, we show that eliminating the future risk is also sufficient to eliminate the current risk, when it is combined with sufficiently small group sizes. In a finite partition, the last group does not face any future risk. We will show that a sufficiently diffused policy eliminates the current risk for the last group and subsequently for all the preceding groups.

2.2 Main Result

The principal wants to maximize the probability that her preferred regime succeeds. To this end, she tries to minimize θ^*_1. We assume that the principal cannot credibly promise payoff incentives for not attacking the PPR. Instead, similar to an information designer, she provides additional information to convince the agents not to attack a viable PPR. The first best outcome is achieved when $\theta_1^* = 0$. We only focus on a specific policy called diffusing coordination risk, $(T, (\alpha))$. By design, if the agents follow the principal's recommendations, then this first best is achieved. As discussed in Section 1.2, when the principal abandons the PPR and recommends the agents to attack the PPR, the agents learn that the PPR cannot succeed and so they will surely follow the recommendation. The challenge is to convince the agents not to attack the PPR.

We have already shown that when the principal diffuses the coordination risk, one possible equilibrium is that the agents follow the principal's recommendation and do not attack the PPR. Thus, the first best is always a possibility. However, it is also possible that the agents are not convinced and attack the PPR. The following theorem shows that if the principal makes each group size sufficiently small, then she can definitely convince the agents not to attack the PPR. We will refer to this policy as a sufficiently diffused policy.

Theorem 1 *Suppose that agents have uninformative prior and that distribution of noise satisfies log-concavity.*[13] *Given (p, τ), there exists $\alpha^* > 0$, such that if the principal sufficiently diffuses the coordination risk - i.e., takes a policy $(T, (\alpha))$ such that $\alpha_t \leq \alpha^*$ for all $t \in \{1, 2, \ldots, T\}$ - then there is a unique equilibrium in which $\theta_1^*(T, (\alpha)) = 0$.*

This implies that, for any non-negative realization of θ - i.e., a viable PPR, there is a uniform bound on how much the principal needs to diffuse the coordination risk in order to make sure that the PPR succeeds in the end.

The key argument behind this main result involves two steps: (1) We argue that a sufficiently small group of agents can be convinced not to attack the PPR when they learn PNV, provided they believe that the agents in the subsequent groups will also follow the principal's recommendation - i.e., there is no future risk. (2) Given finite partition, the last group does not face any future risk - i.e., $\theta_{T+1}^* = 0$ and so can be convinced not to attack the PPR if the group size is sufficiently small. Then the same holds for the penultimate group and so on. Thus, the coordination risk unravels from the end.[14] In this section, we establish this formal inductive argument. We show that for

[13] The uninformative prior assumption is only for simplicity. In Section 4 we will show that the result is robust to informative prior. Log-concavity is sufficient for the global game argument of iterated elimination of never best responses, when there is public information of continued viability. Angeletos et al. (2007) use similar public information and assume normal distribution, which is indeed log-concave.

[14] Huang (2017) shows a similar result where a sufficiently patient long run principal can dissuade agents from attacking the currency regardless of the actual strength. The mechanism, however, is very different. The author

any t, if the coordination risk has unraveled until $(t+1)$ - i.e., $\theta^*_{t+1} = 0$ and $\alpha_t < \alpha^*$, then the coordination risk also unravels until t - i.e., $\theta^*_t = 0$.

To understand the inductive argument, consider the game at t and $\theta^*_{t+1} = 0$. Recall from Proposition 2 that there is always an equilibrium in which $\theta^*_{t+1} = 0$ - i.e., all the agents follow the principal's recommendation from group $(t+1)$ onwards. Given this unraveling of coordination risk until $(t+1)$, the agents in group t only face current coordination risk - i.e., the strategic uncertainty is from other agents in their own group. The agents publicly learn $\theta_1 \geq h_t(0)$ and they have their noisy private information. The public news implies that the initial strength θ_1 is large enough such that the PPR has sustained all the attacks until now. From Proposition 2 we can say that (θ^*_t, s^*_t) are such that either $\theta^*_t = 0$ or

$$\frac{\frac{\theta^*_t}{\alpha_t}}{F\left(F^{-1}\left(\frac{\theta^*_t}{\alpha_t}\right) + \sqrt{\tau}(h_t(\theta^*_t) - h_t(0))\right)} = p \quad , \tag{8}$$

and

$$s^*_t = h_t(\theta^*_t) + \frac{1}{\sqrt{\tau}} F^{-1}\left(\frac{\theta^*_t}{\alpha_t}\right). \tag{9}$$

The inductive argument claims that $\theta^*_t = 0$ is the only equilibrium. This implies that in the absence of future coordination risk, agents in group t will ignore their private information and follow the principal's recommendation. However, note that the PNV is endogenous and it influences the strategic uncertainty differently across groups - i.e., the $h_t(.)$ function differs across groups in the above two equations. Assuming no future risk, in Step 1 we show that agents in group 1 ignore their private information when $\alpha_1 < \alpha^*$ (Proposition 3) and then in Step 2, we show that the argument holds for any group t (Proposition 4). The final step combines the two propositions and shows that the coordination risk unravels from the end.

2.2.1 Step 1: Continued viability and Current Coordination Risk

In this section, we show that the inductive argument holds true for group 1: if the coordination risk has unraveled until group 2 and α_1 is sufficiently small, then the coordination risk will unravel

argues that the principal will eventually build sufficient reputation and then agents will stop attacking. However, if there are only a finite number of attacks, then the principal would have defended against the last attack regardless of the fundamental. Note that this unraveling mechanism in Huang (2017) crucially depends on the principal having future concern, while we are considering a one-shot game for the principal. In our asynchronous coordination game, it is the agents who have dynamic concerns but not the principal.

until group 1. If θ_1 is the fundamental strength and there is no future risk, then the PPR succeeds iff $w_1 < \theta_1/\alpha_1$. Hence, the effective fundamental strength is θ_1/α_1. Recall from Proposition 1 that in the absence of PNV, the threshold fundamental is p. Thus, in equilibrium the PPR succeeds if $\theta_1/\alpha_1 \geq \theta_1^*/\alpha_1 = p$. Since $\theta_1^* = \alpha_1 p$, the coordination risk goes down proportionally with α_1.

If the principal does not abandon the PPR, the agents learn PNV. This information is asymmetric in nature. The agents learn that the PPR continues to be viable but learn nothing about the viability of the other regime. In equilibrium, this will make the agents more optimistic about the success of the PPR. Of course, the extent of the influence of PNV depends on the precision of their private signals. We argue that if the signals are sufficiently noisy, then agents ignore their private information and follow the principal's recommendation (See Lemma 2 in the Appendix B). Given the signal structure s and (α) commonly known, the private information about the effective fundamental strength θ_1/α_1 is

$$\frac{s}{\alpha_1} = \frac{\theta_1}{\alpha_1} + \frac{\sigma}{\alpha_1}\epsilon.$$

Thus, the scale parameter σ/α_1 is inversely proportional to α_1. By choosing a sufficiently small α_1, the principal can make the effective private information sufficiently noisy. This gives us the following proposition.

Proposition 3 *Suppose that agents in group* 1 *publicly learn about the viability of the PPR and they do not face any future risk - i.e.,* $\theta_2^* = 0$. *There exists* $\alpha^* > 0$ *such that if* $\alpha_1 < \alpha^*$, *then agents will ignore their private information and never attack the PPR.*

For group $t = 1$, Equation (8) captures the belief of threshold agent s_1^* that the PPR will succeed. The denominator on the LHS captures the influence of PNV, $P(\theta_1 \geq 0 | s = s_1^*)$. For group 1, $h_1(x) = x$. Imagine that $\alpha_1 = 1$. Then the game is the static regime change game as in Section 1.1 with the modification that agents learn PNV. Given the precision of private signals τ, let $G(\theta_1^*, \sqrt{\tau})$ be the belief of the threshold agent s_1^* that $\theta_1 \geq \theta_1^*$. Then,

$$G(\theta_1^*, \sqrt{\tau}) := \frac{\theta_1^*}{F(F^{-1}(\theta_1^*) + \sqrt{\tau}\theta_1^*)}. \tag{10}$$

Figure 3 shows this belief, $G(\theta_1^*, \sqrt{\tau})$ for any threshold θ_1^*. In the absence of PNV, for any possible θ_1^*, the threshold agent with private information $s_1^*(\theta_1^*)$ believes that the probability that

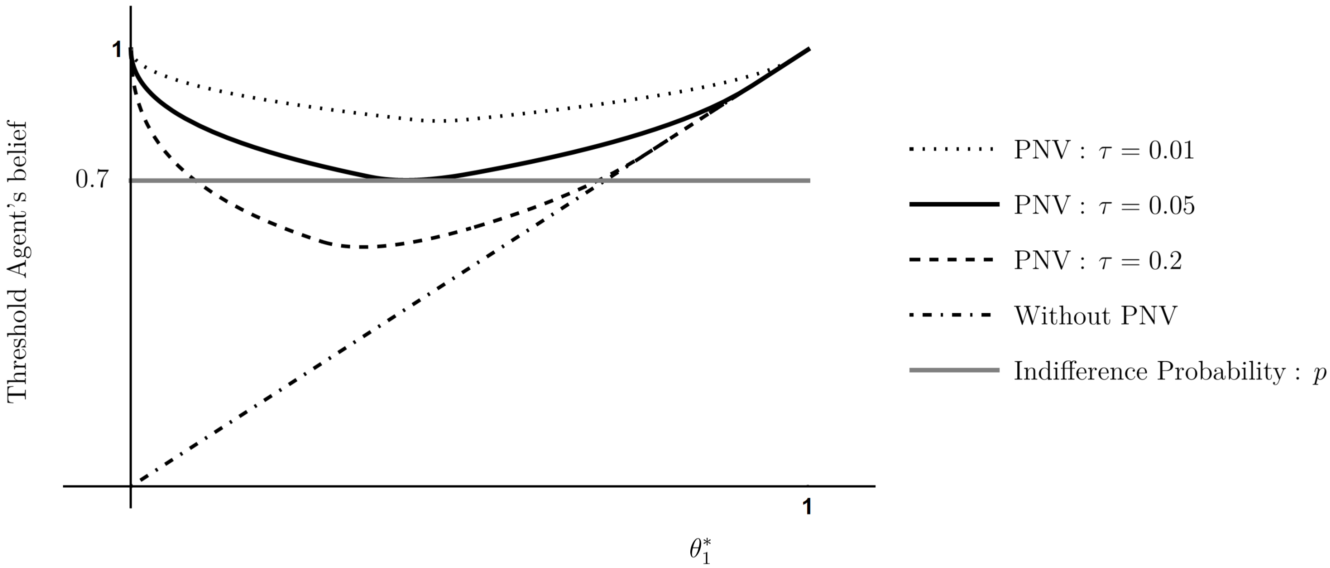

Figure 3: $G(\theta_1^*, \sqrt{\tau})$ when $f(x) = (2+4x)\mathbf{1}(-0.5 \leq x < 0) + (2-4x)\mathbf{1}(0 \leq x \leq 0.5)$

the PPR will succeed is $P(\theta_1 \geq \theta_1^* | s_1^*(\theta_1^*)) = \theta_1^*$. This is the 45° line in Figure 3. In equilibrium, the threshold agent believes that the PPR will succeed with probability p. Hence, we have a unique equilibrium $\theta_1^* = p$. When agents learn PNV, the threshold agent's belief is $P(\theta_1 \geq \theta_1^* | \theta_1 \geq 0, s_1^*(\theta_1^*)) = G(\theta_1^*, \sqrt{\tau})$, which is a non-monotonic function of θ_1^*. As can be seen from Figure 3, if τ decreases, the curve of $G(\theta_1^*, \sqrt{\tau})$ shifts upwards. When τ is sufficiently small, there is no positive θ_1^* such that $G(\theta_1^*, \sqrt{\tau}) = p$. Thus, $\theta_1^* = 0$ is the unique monotone equilibrium fundamental threshold.

When only α_1 share of agents are playing the static regime change game, we can see from Equation (8), the belief of the threshold agent that the PPR will succeed is

$$\frac{\frac{\theta_1^*}{\alpha_1}}{F\left(F^{-1}\left(\frac{\theta_1^*}{\alpha_1}\right) + \alpha_1\sqrt{\tau}\frac{\theta_1^*}{\alpha_1}\right)} = G(\frac{\theta_1^*}{\alpha_1}, \alpha_1\sqrt{\tau}). \tag{11}$$

Although the principal cannot change τ, by choosing a sufficiently small α_1 she can make the effective precision $\alpha_1\sqrt{\tau}$ sufficiently small. This induces the agents to ignore their private information and follow her recommendation.

2.2.2 Step 2: Maximum Current Risk Across Groups

In this section, we will show that if sufficient diffusion eliminates the current coordination risk in group 1 (as in Step 1), then it will do so for the later groups as well. Consider any group t and suppose that the coordination risk has unraveled after group t - i.e., $\theta^*_{t+1} = 0$. This implies that the agents in group t are facing only the current coordination risk. The PNV is endogenously generated depending on how aggressively agents have attacked the PPR. The effect of this news on the strategic uncertainty differs across groups. As the game reaches a later group, the agents learn that the PPR has survived attacks from a larger mass of agents. If agents are not facing any future risk, in any equilibrium, the agents in later groups will be more optimistic about the success of the PPR. In this sense, the first group of agents are least influenced by PNV. If the fundamental strength starts with $h_t(x)$, then at t, the fundamental strength that remains is x. Therefore, $h_t(x) \geq x$ and $(h_t(x) - x)$ captures the attack until group $(t-1)$ when the fundamental strength starts with $h_t(x)$. Imagine that the initial fundamental strength is $h_t(0)$ versus $h_t(x)$ for some $x > 0$. In any monotone equilibrium, there is more attack if the initial strength is $h_t(0)$ as opposed to $h_t(x)$. Therefore, $h_t(x) - x \leq h_t(0) - 0$.

Lemma 1 *In any possible monotone equilibrium, for all* $t \in \{1, 2, \ldots, T\}$ *and any* $x \geq 0$, $h_t(x) - h_t(0) \leq x$.

The result follows directly from the arguments preceding the lemma. Given any equilibrium, the threshold agent believes that the PPR will succeed with probability (as in Equation (8))

$$P\left(\frac{\theta_t}{\alpha_t} \geq \frac{\theta^*_t}{\alpha_t} \middle| s^*_t, \frac{\theta_t}{\alpha_t} \geq 0\right) = \frac{\frac{\theta^*_t}{\alpha_t}}{F\left(F^{-1}\left(\frac{\theta^*_t}{\alpha_t}\right) + \sqrt{\tau}(h_t(\theta^*_t) - h_t(0))\right)}$$
$$\geq \frac{\frac{\theta^*_t}{\alpha_t}}{F\left(F^{-1}\left(\frac{\theta^*_t}{\alpha_t}\right) + \alpha_t\sqrt{\tau}\frac{\theta^*_t}{\alpha_t}\right)}, \text{ Using Lemma 1.}$$

Substituting $G(.)$ as defined in Equation (11), we get the following inequality

$$P\left(\frac{\theta_t}{\alpha_t} \geq \frac{\theta^*_t}{\alpha_t} \middle| s^*_t, \frac{\theta_t}{\alpha_t} \geq 0\right) \geq G\left(\frac{\theta^*_t}{\alpha_t}, \alpha_t\sqrt{\tau}\right). \tag{12}$$

Recall that for any threshold x, $G(x, \alpha_t\sqrt{\tau})$ captures the belief of the threshold agent in group 1 that the PPR will succeed, when there is no future risk and the group size is α_t. Thus, for the same group size $\alpha_t = \alpha_1$, if $G(x, \alpha_1\sqrt{\tau}) > p$ for all x, then $P\left(\theta_t/\alpha_t \geq x \,|s^*_t(x), \theta_t/\alpha_t \geq 0\right) > p$ for all x. Therefore, if a small group size can eliminate the current coordination risk in group 1, then the same group size eliminates the current coordination risk in later groups as well.

Proposition 4 *Suppose that agents in group t publicly learn about the continued viability of the PPR and that they do not face any future risk - i.e., $\theta^*_{t+1} = 0$. If $\alpha_t < \alpha^*$ then agents will ignore their private information and never attack the PPR.*

The result follows directly from the arguments made preceding the proposition. This shows that the inductive argument holds for any group t.

2.2.3 Step 3: Unraveling of Coordination Risk from the End

Proposition 3 says that the inductive argument holds for group 1 if $\alpha_1 < \alpha^*$ and Proposition 4 says that given the same group size, if the inductive argument holds for group 1, then it also holds for any $t > 1$. Given $\theta^*_{T+1} = 0$, the above two propositions then imply that if $\alpha_t < \alpha^*$ for all t, then agents in the last group do not attack the PPR. Knowing this, agents in the penultimate group do not attack the PPR either and so on until group 1. Thus, by sufficiently diffusing the coordination risk:

1. The principal convinces the agent in any group t that the agents in the subsequent groups will follow her recommendation and

2. This makes the news of continued viability sufficiently strong to convince the agents in group t not to attack the PPR.

Consequently, no agent attacks a viable PPR.

2.3 Discussion

We consider a mass of agents who take decisions based on their private information. These agents would like to coordinate but cannot communicate. If the agents can be assured that other agents are taking a specific action, then they will take the same action. Through diffusing the coordination risk, the principal is facilitating this assurance. When the agents learn about the PNV and are assured that the agents in the subsequent groups will follow the recommendation, they follow the recommendation as well. The main result shows that a sufficiently diffused policy leads to a unique equilibrium in which all agents follow the principal's recommendation and do not attack a viable

PPR. The unique equilibrium implies that it is not only possible that a viable PPR will succeed in the end, rather it will always succeed. However, we only claim that this is one possible way to eliminate the coordination risk. We do not consider a generalized policy space.

The proof exploits the influence of PNV when agents in a group are assured that agents in subsequent groups will follow the principal's recommendation - i.e., there is no future risk. This is possible because from Proposition 2 we know that there is always an equilibrium in which there is no future risk after some group t. However, if the principal fails to provide such an assurance, then the actions of agents in subsequent groups depend on the actions of the agents in group t. Given the vast multiplicity of equilibria, we cannot be certain whether higher diffusion will reduce coordination risk or θ_1^*. Numerically, for $p = 0.7$, $\tau = 10$ and $f(x) = (2+4x)\mathbf{1}(-0.5 \leq x < 0)+(2-4x)\mathbf{1}(0 \leq x \leq 0.5)$, we find that with $(T, (\alpha)) = (3, (0.3, 0.35, 0.35))$, there is an equilibrium with $\theta_1^* = 0.5388$, while with $(T, (\alpha)) = (4, (0.3, 0.35, 0.05, 0.3))$, there is an equilibrium with $\theta_1^* = 0.5892$. Thus, higher diffusion may lead to an increase in θ_1^*. In the Online Appendix, we show the numerical result for uniform diffusion $(T, (\alpha))$ where $\alpha_t = 1/T$ to illustrate that for a sufficiently diffused policy $(T, (\alpha))$, $\theta_1^* = 0$ is the unique equilibrium.

In order to convince the agents not to attack the PPR, the principal needs to overcome two hurdles. First, agents' reluctance to favor the PPR which is captured by the payoff parameter, p and second, the precision of their private information which is captured by τ. Recall that agents do not attack the PPR if they believe that it will succeed with a probability of at least $p = 1/\left(1 + \frac{b_1 - c_1}{b_0 - c_0}\right)$. Suppose that the net benefit from attacking the PPR- $(b_0 - c_0)$ is higher, or the net benefit from favoring the PPR- $(b_1 - c_1)$ is lower, then p is higher. Hence, the agents are more reluctant to favor the PPR. We expect that convincing such agents not to attack the PPR is more difficult. Also, if τ is higher, we expect that it will be harder to convince the agents to ignore their private information and follow the principal's recommendation. The following Corollary confirms this intuition.

Corollary 2 *The critical mass α^* that eliminates the coordination risk is decreasing in p and τ.*

If agent's actions are divisible, then coordination risk can also be diffused through setting a ceiling on individual attack. Consider a slightly different setup. Suppose that the game lasts for unit time interval $[0, 1]$. An agent can partially attack the PPR at any time between $[0, 1]$. The maximum attack an agent can make is 1. The principal restricts individual attack in any time interval and hence, restricts the aggregate attack. Consider, for example, the case in which investors decide whether to withdraw their investments. The principal sets a limit on how much they can withdraw in any time interval. Thus, a policy $(T,(\alpha_1,\alpha_2,\ldots,\alpha_T))$ is equivalent to saying that the principal restricts the maximum withdrawal that an agent can make in the time interval of $[0,\alpha_1]$ to a share α_1 of the total funds, with the maximum withdrawal in the interval $(\alpha_1,\alpha_1+\alpha_2]$ equal to α_2, and so on.

3 Selected Applications

For standard coordination games of regime change, we have shown that the policy of diffusion can be effective in averting coordination failure. Our theory can be easily applied to many well-known coordination problems. In this section, we bring our theory to very specific economic environments to better understand the feasibility and effectiveness of this policy.

3.1 Panic-Based Runs

Consider a borrower trying to persuade her creditors to roll over their debts. If all the debts were maturing at the same time, then the creditors face the strategic uncertainty whether others can be convinced to rollover or not. Many viable borrowers may fail to convince the investors to roll over their debts. Brunnermeier (2009) argues that this coordination risk in maturity mismatch is one of the main causes of the recent financial crisis. In reality, diffused policies that prevent creditors running at the same time are commonly observed for financial firms performing liquidity transformation and thus facing maturity mismatch problems (See He and Xiong (2012)). Choi et al. (2016) show that corporate bond issuers adopt diversified debt rollovers across maturity dates.[15]

Suppose a borrower who has adopted an asynchronous debt structure $(T,(\alpha))$ in which α_t share of short-term debts mature at time $t\in\{1,2,\ldots T\}$ is in distress. In total, there is mass 1 of creditors - i.e., $\sum_{t=1}^{T}\alpha_t=1$ - who lend \$1 to finance an investment. Assume that, after the long term project matures, the realized return R is sufficiently high to pay all possible claims. So, the

[15] In practice, hedge fund managers can lift investor-level gates to limit investors' redemptions within a certain period. A common investor-level gate limits redemptions to 25% of an investor's money each quarter over four quarters (see "Hedge funds try new way to avoid big redemptions," by Alistair Barr (June 10, 2010), http://www.marketwatch.com/story/hedge-funds-try-new-way-to-avoid-big-redemptions-2010-06-10). In the market of Money Market Mutual Funds (MMMF), the recent Securities and Exchange Commission reform of MMMF in the U.S. provides fund managers with the ability to set redemption gates within a certain period when a fund's liquidity position is not favorable (see http://www.sec.gov/rules/final/2014/33-9616.pdf).

financial distress ends upon the maturity of long-term investment.

At time 0, the borrower has liquid assets to sustain $\theta \geq 0$ share of withdrawals. As before, w_t stands for the proportion of time t creditors who decide to withdraw their debt. For convenience, let us define $w^t = \sum_{u=t+1}^{T} \alpha_u w_u$ as the total withdrawal after t and $w^T = 0$. At t, the remaining liquidity is θ_t. If $\theta_t < 0$, then the borrower has already defaulted and the creditors who are yet to make decisions will get 0 regardless of their actions. If the borrower has not yet defaulted, then creditors of debts maturing at time t decide whether to withdraw ($a_i = 0$), or roll over the debt ($a_i = 1$) till maturity of the project. If a creditor withdraws, he gets 1 if θ_t can sustain the current withdrawal $\alpha_t w_t$. If $\theta_t < \alpha_t w_t$, then the borrower defaults at t. For simplicity, we assume that the long-term investment has no liquidation value before it matures. The withdrawing creditors will split the liquid asset θ_t. Thus, for $\theta_t \geq 0$, the payoff from withdrawal is

$$u(0, \theta_t, \alpha_t w_t, w^t) = \begin{cases} 1 & \text{if } \theta_t \geq \alpha_t w_t \\ \frac{\theta_t}{\alpha_t w_t} & \text{if } \theta_t < \alpha_t w_t. \end{cases}$$

On the other hand, if the creditor rolls over until the maturity of the project, then he gets $1+r < R$ if the projects succeeds in the end and 0 otherwise.

$$u(1, \theta_t, \alpha_t w_t, w^t) = \begin{cases} 1+r & \text{if } \theta_t \geq \alpha_t w_t + w^t \\ 0 & \text{if } \theta_t < \alpha_t w_t + w^t. \end{cases}$$

This game of dynamic run demonstrates two important differences in the payoff structure compared to Section 1.2. First, although the payoff from rollover only depends on whether the borrower defaults or not in the end and thus, depends on depositors' current and future actions ($\alpha_t w_t + w^t$), the payoff from withdrawing depends only on the current actions ($\alpha_t w_t$). This asymmetry in payoff creates a higher incentive for depositors to withdraw. Second, when the borrower defaults, the payoff from withdrawing is not constant but dependent on the level of aggregate withdrawal as well as the residual fundamental.[16] In the following proposition we show that despite these differences

[16] Morris and Shin (2004) consider a similar debt run game in which the creditors make their decisions simultaneously. The payoff structure in this model is similar to the one in Goldstein and Pauzner (2005), where creditors' payoff from withdrawing is negatively dependent on aggregate withdrawal. Goldstein and Pauzner (2005) show that the global game perturbation uniquely selects a monotone equilibrium even with the lack of global strategic complementarity.

a sufficiently asynchronous debt structure makes a borrower immune to panic-based runs.

Proposition 5 *There exists some $\alpha^* > 0$ such that if the debt structure $(T, (\alpha))$ satisfies $\alpha_t < \alpha^*$ for all $t \in \{1, 2, \ldots, T\}$, then there is no panic-based debt run - i.e., $\theta_1^* = 0$.*

We extend our result to incorporate any bounded payoff with strategic complementarity. Since, under sufficient diffusion, future risk is eliminated ($w^t = 0$), the payoff from withdrawing and rolling over for time t creditors depends only on whether the borrower can successfully sustain the current withdrawals. Thus, the asymmetric payoff structure is compatible with our theory. Different from our benchmark model, one may also be interested in the creditors' decision to lend in the first place. Given that sufficient diffusion eliminates the coordination risk, the creditors have more incentive to lend to the borrower. However, if the borrower undertakes such policy only in times of distress, this argument does not apply.

3.2 Tax Havens

Tax haven countries attract offshore investments through nominal taxation and secrecy of tax evading investors. Such tax sheltering activities pose a serious challenge to other jurisdictions. To counter this loss of valuable public revenue, other sovereign countries and supranational organizations, e.g. Organization for Economic Cooperation and Development(OECD) have initiated several policies to fight against the tax haven. The international pressure makes it difficult for tax havens to maintain the status quo.

Following Konrad and Stolper (2016), this can be captured in a regime change game. A continuum of opportunistic foreign investors decide whether to move their capital to this tax haven country ($a_i = 0$) or not ($a_i = 1$). Such a move is profitable only if the tax haven can maintain the secrecy regime (regime 0). However, if the tax haven does not get sufficient investment (w), then it will not be generating enough profit to withstand the international pressure (θ). Following the standard global game argument, we can say that the tax haven country abandons the status quo and exchanges information with other countries if and only if the international pressure is sufficiently high ($\theta \geq \theta^*$).

Imagine an organization like OECD as our principal. Suppose this organization checks in every α^* interval of time whether a transparent regime is still viable. That is, whether the tax haven country is already making sufficient profit that it will never abandon the secrecy regime despite no further investment. If the transparent regime continues to be viable, the organization continues exerting pressure and abandons otherwise. Following such a policy, our principal can successfully convince all opportunistic investors not to invest their capital in the tax haven country and a viable transparent regime can be implemented regardless of the international pressure (however small).

On the other hand, if the tax haven country is our principal, she can adopt the opposite policy. She checks in every α^* interval of time whether she can still maintain the secrecy regime, otherwise abandon the status quo. Following a sufficiently diffused policy, the tax haven country can always defend a viable secrecy regime regardless of the international pressure (however large).

3.3 Currency Attack

The People's Bank of China imposes a ceiling on the conversion of Renminbi to foreign exchange assets by individuals or institutions.[17] China's policies have been successful in maintaining a relatively stable exchange rate over a period of decades. From the perspective of deterring currency attacks, this paper provides a rationale for such a policy. Similar to Morris and Shin (1998), consider a mass of speculative attackers deciding whether to convert the domestic currency into foreign currency ($a_i = 0$) or not ($a_i = 1$). If the central bank does not have enough foreign exchange reserves (θ) to withstand the speculative attack (w), then the central bank cannot maintain the current exchange rate and has to devalue the domestic currency. If the currency is devalued, then the attackers can buy back the domestic currency later at a lower rate and make a profit. However, if the central bank does not devalue, then attacking the currency is costly. Thus, speculators attack the domestic currency when they believe other speculators will do the same. When a ceiling on currency conversion is imposed, the fact that the domestic currency has not been devalued yet works as PNV. If the ceiling is sufficiently small, then PNV convinces the speculators that there will be no future attack and they decide not to attack.

[17] For example, Chinese residents may purchase up to $\$50,000$ worth of foreign exchange per year (See Neely (2017)).

3.4 Network Externality

When a new technology or new standard arrives, it is not likely to get adopted en masse by firms in an industry. Each firm gets higher benefit from adopting the new technology when other firms also adopt the same. This is referred to as "network externality" (See Katz and Shapiro (1986) and Farrell and Saloner (1985)). For example, after Automated Clearing House (ACH) electronic payments system was introduced, individual banks adopted it over time. Gowrisankaran and Stavins (2004) identify the extent of network externalities using a quarterly panel data set on individual bank adoption and usage of ACH. If a principal wants a viable new standard to be successfully adopted, she can approach the firms in small groups. If the firms know that the principal does not try to persuade them to adopt a non-viable new standard, they will follow the principal's recommendation and adopt the new standard.

3.5 Divide and Rule

Divide and Rule is a common political strategy that rulers use to dissuade agents from revolting. By making a division, the ruler convinces one group that the other group is not going to attack the incumbent regime. This stops either group from attacking. This policy has a general appeal for understanding strategic uncertainty (See Segal (2003)) beyond the specific problem of a ruler trying to remain in power. Acemoglu et al. (2004) suggest a division rule of promising transfers to the non-attackers by taxing the attackers. This policy, although simple and appealing, may not always be feasible in the general context. For example, imagine a political regime change game similar to Edmond (2013). Suppose that an alternative leader is trying to persuade the agents to revolt against the status quo. It does not seem likely that she can devise a rule to reward the attackers and punish the agents who do not attack. Or, consider an organizer persuading people to attend a conference or vote on a specific issue. The division rule that we propose does not need the principal to have the authority to change payoffs. She only informs the subsequent groups about the survival of PPR. For a small group, such news can be overwhelming to convince them not to attack the PPR, provided they believe that the subsequent groups are not attacking. Thus, a sufficiently diffused policy guarantees that the last group does not attack and so does the penultimate group and so on. Moriya and Yamashita (2015) and Inostroza and Pavan (2017) argue that a principal may achieve a better outcome by asymmetrically disclosing information. Although our principal discloses the information publicly, the asymmetry follows from the sequentiality and irreversibility of actions. An agent in group t cannot act on the information disclosed at a later date $t' > t$.

4 Extensions

4.1 Accumulated Information

Suppose that agents receive private information regarding the past attacks as the principal diffuses the coordination risk. One can suspect that agents will no longer ignore their private information as it becomes more informative. We argue that our main result is robust to such accumulated information. To incorporate such additional information in a tractable way, we will assume that F is normal distribution and the prior is uniform over the real line. Although unconditional probabilities are not well defined for such a distribution, we will assume that the principal minimizes the coordination risk by minimizing the threshold θ_1^*. After group $(t-1)$ agents make their decisions, the rest of the agents privately receive noisy information s_t about the proportion of them that attacked - i.e., w_{t-1}, where

$$s_t = \Phi^{-1}(w_{t-1}) + \frac{1}{\sqrt{\tau_t}}\epsilon_t.$$

We will assume that ϵ_t is i.i.d standard normal. This particular functional form, first suggested by Dasgupta (2007), is assumed for tractability.

We focus on monotone equilibrium. Note that agents in group t receive multiple private signals - i.e., $s_1, s_2, \ldots s_t$. Let $\zeta_t(s_1, s_2, \ldots s_t)$ be a sufficient statistic for the aggregate information. The monotone equilibrium is characterized by the thresholds (θ_t^*, ζ_t^*) - i.e., agents in group t attack the preferred regime iff $\zeta_t < \zeta_t^*$, and the regime survives all the attacks from t onwards if $\theta_t \geq \theta_t^*$.

Under this Gaussian error structure $\theta|(s_1, s_2, \ldots s_t) \sim N(\zeta_t, 1/z_t)$. The sufficient statistic at t, $\zeta_t(s_1, s_2, \ldots s_t)$, and the precision of the aggregate information at t, z_t, can be recursively defined as $\zeta_1(s_1) = s_1$, $z_1 = \tau_1$, while,

$$\begin{aligned}
\zeta_{t+1} =& \frac{z_t}{z_t + \tau_{t+1} z_t}\zeta_t + \frac{\tau_{t+1} z_t}{z_t + \tau_{t+1} z_t}\left(\zeta_t^* - \frac{1}{\sqrt{z_t}} s_{t+1}\right) \\
z_{t+1} =& z_t(1 + \tau_{t+1}).
\end{aligned}$$

Proposition 6 *In the game with agents receiving noisy private information about past attacks, if* *the precision of the aggregate information - z_T does not converge to infinity at a faster rate than T^2 - i.e.,* $\lim_{T\to\infty}\sqrt{z_T}/T \to 0$*, then sufficient diffusion eliminates all the coordination risk.*

We have mentioned before that Huang (2017) shows a similar unraveling result when a principal has dynamic reputation concern. The author also shows that the unraveling does not hold if the speculators learn at a faster speed than usual. However, unlike the dynamic setup in Huang (2017), this game is a one-shot game for the principal. The dynamics in this model come from the partition of the mass of agents. Since the mass of agents is fixed, and the source of the additional information is the past action of a fraction of these mass of agents, it is reasonable to think that there is some upper bound to the precision of private information. Then the above limiting condition is surely satisfied. Consider two specific examples: (1) $\tau_t = \gamma^{\alpha_{t-1}} - 1$ or (2) $\tau_t = \alpha_{t-1} \times \beta$. Both these examples illustrate that if agents learn about past attacks from a smaller share of agents, then the precision of such a signal is less. It can be easily checked that the limiting condition holds. Thus, sufficient diffusion eliminates the coordination risk.

4.2 Informative Prior

Throughout this paper, we assume that agents have an uninformative prior. If agents have some informative prior belief about θ_1, then this could weaken the impact of PNV. To posit this issue in a tractable way, we consider a Gaussian world. We show that the main result is robust to such changes.

Proposition 7 *In the game with a Guassian prior $\theta \sim N(\theta_0, 1/\tau_0)$ and private information $s = \theta + \epsilon/\sqrt{\tau_1}$, where $\epsilon \sim N(0,1)$, sufficient diffusion eliminates all the coordination risk.*

In the absence of PNV, Morris and Shin (2004) and Hellwig (2002) (See Theorem 1 in either paper) show that if $\tau_0/\sqrt{\tau_1}$ is sufficiently small, then there is a unique equilibrium. Recall that by diffusing the coordination risk, we scale down the precision of the private information τ to $\alpha^2\tau$. This also applies to the prior belief in the sense that τ_0 is scaled down to $\alpha^2\tau_0$. Therefore, as α becomes small, $\tau_0/\sqrt{\tau_1}$ will eventually become sufficiently small. Thus, under sufficient diffusion, the uniqueness of equilibrium condition is satisfied. However, since agents learn PNV, a very different equilibrium is selected.

5 Conclusion

This paper proposes a simple policy, called diffusing coordination risk, to eliminate the strategic uncertainty in a global game of regime change. A principal diffuses the coordination risk by making a partition of the mass of agents. She recommends the agents in each group not to attack her preferred regime in a chronological fashion. However, she abandons her preferred regime when she realizes that it cannot succeed. We show that when the principal sufficiently diffuses the coordination risk, agents ignore their private information and follow the principal's recommendation.

Conventional wisdom suggests that sequential moves can help agents to coordinate better. However, the novelty of the result is that all the coordination risk can be eliminated. From a methodological perspective, our paper develops an inductive argument to show that the coordination risk unravels from the end. Note that this is not a standard backward induction argument since we model a world with incomplete information.

From an applied perspective, we show that our policy can be easily adopted for addressing prevalent coordination problems such as eliminating the risk of debt run, currency crisis etc. Keeping this in mind, we extend our model to accommodate certain features that were absent in the benchmark model but may be important for applications. The scope of these extensions, however, are limited to changes in payoff structure, additional private information about past decisions or prior belief. Within the scope of this paper we refrain from discussing the effectiveness of limited diffusion (if diffusion is costly), heterogeneity (which agents should move early?), or the possible signaling effect of the policy (such as when the borrower adopts this policy under distress). We think these are promising extensions for future research.

Appendix

Appendix A: Monotone Equilibrium

Proposition 1 is an existing result, and its proof can be found in Morris and Shin (2003). For completeness, we reproduce the proof in the Online Appendix.

Proof of Proposition 2 An agent in group t who receives private information s about $\theta = \theta_1$ and learns PNV, believes that the PPR will succeed with probability $P(\theta_t \geq \theta_t^* | s, \theta_t \geq 0)$. The residual strength θ_t evolves endogenously. For any threshold θ_1^*, $h_t(.)$ maps this residual strength to the initial strength θ_1. By definition $h_t(\theta_t^*) = \theta_1^*$. Thus,

$$P(\theta_t \geq \theta_t^* | s, \theta_t \geq 0) = P(\theta_1 \geq h_t(\theta_t^*) | s, \theta_1 \geq h_t(0)) = \frac{F(\sqrt{\tau}(s - h_t(\theta_t^*)))}{F(\sqrt{\tau}(s - h_t(0)))}. \tag{13}$$

Given the equilibrium threshold θ_1^*, and the resulting $h_t(.)$, let $s_t^*(\theta_1^*)$ be the threshold of private information such that agents in group t do not attack the preferred regime if and only if $s \geq s_t^*(\theta_1^*)$. If agents in group t are following this monotone strategy, then for $\theta_1 \geq \theta_1^*$ or equivalently for $\theta_t \geq \theta_t^*$, the aggregate attack from group t will leave sufficient residual strength to survive thereafter - i.e., $\theta_{t+1} \geq \theta_{t+1}^*$. Thus,

$$\theta_t - \alpha_t w_t(\theta_1) \geq \theta_{t+1}^* \quad \text{iff } \theta_1 \geq \theta_1^*,$$
$$\Rightarrow \; w_t(\theta_1^*) = F(\sqrt{\tau}(s_t^*(\theta_1^*) - \theta_1^*)) = (\theta_t^* - \theta_{t+1}^*)/\alpha_t.$$

Hence, the threshold signal s_t^* must satisfy Equation (7):

$$s_t^*(\theta_1^*) = \theta_1^* + \frac{1}{\sqrt{\tau}} F^{-1}\left(\frac{\theta_t^* - \theta_{t+1}^*}{\alpha_t}\right).$$

Substituting $s_t^*(\theta_1^*)$ into the Equation (13), we have

$$P(\theta_t \geq \theta_t^* | s_t^*(\theta_1^*), \theta_t \geq 0) = \frac{\frac{\theta_t^* - \theta_{t+1}^*}{\alpha_t}}{F\left(\sqrt{\tau}(\theta_1^* - h_t(0)) + F^{-1}\left(\frac{\theta_t^* - \theta_{t+1}^*}{\alpha_t}\right)\right)}.$$

Recall that an agent does not attack the PPR iff he believes that the PPR will succeed with a probability at least p.

At any time t, by definition, $\theta_t^* \in [\theta_{t+1}^*, \theta_{t+1}^* + \alpha_t]$. Given PNV at t, there is always an equilibrium in which, $\theta_t^* = \theta_{t+1}^* = \ldots = \theta_T^* = 0$. For other possible equilibria, we can consider the following three cases: 1. $P(\theta_t \geq \theta_t^* | s_t^*(\theta_1^*), \theta_t \geq 0) > p$ for all $\theta_t^* \in [\theta_{t+1}^*, \theta_{t+1}^* + \alpha_t]$; 2. $P(\theta_t \geq \theta_t^* | s_t^*(\theta_1^*), \theta_t \geq 0) < p$ for all $\theta_t^* \in [\theta_{t+1}^*, \theta_{t+1}^* + \alpha_t]$; 3. There exists $\theta_t^* \in [\theta_{t+1}^*, \theta_{t+1}^* + \alpha_t]$ such that $P(\theta_t \geq \theta_t^* | s_t^*(\theta_1^*), \theta_t \geq 0) = p$.

Case 1: This case can only occur when $\theta_t^* = \theta_{t+1}^* = \theta_{t+2}^* = ... = \theta_T^* = 0$.

Proof. If $P(\theta_t \geq \theta_t^* | s_t^*(\theta_1^*), \theta_t \geq 0) > p$ for all $\theta_t^* \in [\theta_{t+1}^*, \theta_{t+1}^* + \alpha_t]$, agents at time t will favor the PPR irrespective of their information. Thus, $\theta_t^* = \theta_{t+1}^*$, and $s_t^* = \theta_1^* - 1/(2\sqrt{\tau})$ (from Equation

(7)). Suppose that $\theta_t^* = \theta_{t+1}^* > 0$, then

$$P(\theta_t \geq \theta_t^* | s_t^*(\theta_1^*), \theta_t \geq 0) = \frac{0}{F(\sqrt{\tau}(h_t(\theta_t^*) - h_t(0)) - \frac{1}{2})} = 0 < p,$$

which contradicts $P(\theta_t \geq \theta_t^* | s_t^*(\theta_1^*), \theta_t \geq 0) > p$. Thus, $\theta_t^* = \theta_{t+1}^* = 0$. This implies $\theta_t^* = \theta_{t+1}^* = \theta_{t+2}^* = ... = \theta_T^* = 0$. ■

Case 2: This case cannot be an equilibrium.

Proof. If $P(\theta_t \geq \theta_t^* | s_t^*(\theta_1^*), \theta_t \geq 0) < p$ for all $\theta_t^* \in [\theta_{t+1}^*, \theta_{t+1}^* + \alpha_t]$, then all agents will attack the preferred regime, irrespective of their information. Thus, $\theta_t^* = \theta_{t+1}^* + \alpha_t$. Since $h_t(\theta_t^*) - h_t(0) \geq 0$ as $\theta_t^* \in [0, 1]$ and the function F has support in $[-1/2, 1/2]$, we have

$$P(\theta_t \geq \theta_t^* | s_t^*(\theta_1^*), \theta_t \geq 0) = \frac{1}{F(\sqrt{\tau}(h_t(\theta_t^*) - h_t(0)) + \frac{1}{2})} = 1 > p,$$

which contradicts $P(\theta_t \geq \theta_t^* | s_t^*(\theta_1^*, \theta_{t+1}^*), \theta_t \geq 0) < p$. Proposition 2 presents all possible monotone equilibria in Case 1 and Case 2. ■

Case 3: If $P(\theta_t \geq \theta_t^* | s_t^*(\theta_1^*), \theta_t \geq 0) = p$ has an interior solution θ_t^*, then this immediately implies the recursive relation (6) holds true. Recall that $\theta_t^* = \theta_{t+1}^* = \ldots = \theta_T^* = 0$ is always an equilibrium and it is consistent with this case.

Proof of Corollary 1 When the game reaches time t and the PPR is viable, - i.e., $\theta_1 \geq h_t(0)$ and there is no current risk for group t, - i.e. $h_t(0) = h_{t+1}(0)$, no one in group t attacks PPR even if $\theta_t = 0$. So, we have $s_t^* \leq h_t(0) - 1/(2\sqrt{\tau})$. From Equation (7), we know that $s_t^* \geq h_t(0) - 1/(2\sqrt{\tau})$ because $\theta_1^* = h_t(\theta_t^*) \geq h_t(0)$ and $\theta_t^* \geq \theta_{t+1}^*$.

The above two inequalities of s_t^* tell us that $s_t^* = h_t(0) - 1/(2\sqrt{\tau})$, $\theta_1^* = h_t(0)$ and $\theta_t^* = \theta_{t+1}^*$. Since $\theta_1^* = h_t(\theta_t^*)$ and h_t is a nondecreasing function, we know that $\theta_{t+1}^* = \theta_t^* = 0$. This means that there is no future risk for group t agents.□

Appendix B: Main result

Proof of Proposition 3 We first consider a static regime change game as in Section 1.1. It is publicly known that the PPR is viable - i.e., $\theta \geq 0$. We show that in this static game, as private information becomes sufficiently noisy, the unique rationalizable strategy for any agent is to ignore his private information and never attack the PPR (Lemma 2). Then in a dynamic game without future risk for the first group, we show that if α_1 becomes sufficiently small, the effective private information becomes sufficiently noisy and thus there is no coordination risk among this group of agents.

A Static Game with PNV

Corollary 3 *Multiple equilibria exist in a static regime change game with public information of viability. In particular, there is a monotone equilibrium (θ^*, s^*) such that either $\theta^* = 0$ and $s^* = -1/(2\sqrt{\tau})$ or the following conditions are satisfied:*

$$G(\theta^*, \sqrt{\tau}) = p, \text{ and } s^*(\theta^*) = \theta^* + \frac{1}{\sqrt{\tau}} F^{-1}(\theta^*).$$

Proof. This is only a special case ($T = 1$) of Proposition 2. ■

Let (θ^{*h}, s^{*h}) be the worst monotone equilibrium in the sense that it has the maximum coordination risk. Then,

$$\theta^{*h} := \max\{\theta \in [0,1] | G(\theta, \sqrt{\tau}) \leq p\}. \tag{14}$$

Lemma 2 *Suppose that agents have uninformative priors, F is log-concave and it is publicly known that $\theta \geq 0$. Then, there is a threshold $\bar{s}_\infty$ such that for $s \geq \bar{s}_\infty$, attacking the preferred regime is not rationalizable. Consequently, there is $\bar{\theta}_\infty$ such that the preferred regime succeeds whenever $\theta \geq \bar{\theta}_\infty$. Furthermore :*

1. *$(\bar{\theta}_\infty, \bar{s}_\infty)$ is a monotone equilibrium, and $\bar{\theta}_\infty = \theta^{*h}$.*
2. *There exists τ^* such that if $\tau < \tau^*$, the monotone equilibrium $(\bar{\theta}_\infty = 0,\ \bar{s}_\infty = -1/(2\sqrt{\tau}))$ is the unique equilibrium.*

Proof. The proof is built in three steps. In step 1, we eliminate the dominated strategies iteratively. In step 2, we argue that the worst possible undominated strategy constitutes a monotone equilibrium. In Step 3, we show that for sufficiently small τ, the unique rationalizable strategy is not to attack, regardless of the private information.

The first two steps are based on the usual global game argument of iterated elimination of strictly dominated strategies. The only difference from the standard argument is the influence of the public news of continued viability. It eliminates the lower dominance region ($a_i(s) = 0$ regardless of others' decisions) and potentially generates multiple monotone equilibria. We will introduce the necessary notations and describe the results from these two steps but leave the formal proof to the Online Appendix.

Step 1: Iterated Elimination of Never Best Responses

After n rounds of elimination of never-best responses, let $\bar{s}_n$ be such that, in the worst possible scenario, an agent with public information $\theta \geq 0$ does not attack the PPR only when $s \geq \bar{s}_n$. Let $\bar{\theta}_n$ be such that, in the worst possible scenario, the PPR succeeds only when $\theta \geq \bar{\theta}_n$. Either $\bar{s}_n = -1/(2\sqrt{\tau})$ and $\bar{\theta}_n = 0$, or we recursively define $\bar{s}_{n+1}$ as,

$$P(\theta \geq \bar{\theta}_n | \bar{s}_{n+1}, \theta \geq 0) = \frac{F(\sqrt{\tau}(\bar{s}_{n+1} - \bar{\theta}_n))}{F(\sqrt{\tau}\bar{s}_{n+1})} = p \tag{15}$$

or, $\bar{s}_{n+1} = -1/(2\sqrt{\tau})$ if $F(\sqrt{\tau}(\bar{s}_{n+1} - \bar{\theta}_n))/F(\sqrt{\tau}\bar{s}_{n+1}) > p$ for all $\bar{s}_{n+1} \in [\underline{s}, \bar{s}]$, where $\underline{s} = \underline{\theta} - 1/(2\sqrt{\tau})$ and $\overline{s} = \overline{\theta} + 1/(2\sqrt{\tau})$. Also, define $\bar{\theta}_{n+1}$ as

$$P(s < \bar{s}_{n+1} | \bar{\theta}_{n+1}) = F(\sqrt{\tau}(\bar{s}_{n+1} - \bar{\theta}_{n+1})) = \bar{\theta}_{n+1}. \tag{16}$$

If $\bar{\theta}_n < \bar{\theta}_{n-1}$, through Equation (15), we have $\bar{s}_{n+1} < \bar{s}_n$. From Equation (16), we have $\bar{\theta}_{n+1} < \bar{\theta}_n$. Since $\bar{\theta}_1 < \bar{\theta}_0$ (see Online Appendix), by induction, we get a decreasing sequence $\{\bar{\theta}_n\}_{n=0}^{\infty}$. Let $\bar{\theta}_\infty = \lim_{n\to\infty} \bar{\theta}_n$. Note that either $\bar{\theta}_\infty = 0$ or $\bar{\theta}_\infty > 0$. For any given $\tau > 0$, if $\bar{\theta}_\infty > 0$, then it must satisfy

$$\lim_{n\to\infty} \frac{F(\sqrt{\tau}(\bar{\theta}_{n+1} - \bar{\theta}_n) + F^{-1}(\bar{\theta}_{n+1}))}{F(\sqrt{\tau}\bar{\theta}_{n+1} + F^{-1}(\bar{\theta}_{n+1}))} = \frac{\bar{\theta}_\infty}{F(\sqrt{\tau}\bar{\theta}_\infty + F^{-1}(\bar{\theta}_\infty))} = G(\bar{\theta}_\infty, \sqrt{\tau}) = p, \tag{17}$$

where $G(.)$ is as defined in Equation (10).

Step 2: The Worst Possible Equilibrium

In step 1 we iteratively eliminate $a_i(s) = 0$ for any $s \geq \bar{s}_\infty$. Accordingly, we find a threshold $\overline{\theta}_\infty$ such that the PPR surely succeeds whenever $\theta \geq \overline{\theta}_\infty$. In step 2, we show that this threshold is identical to the worst monotone equilibrium threshold $\theta^{*h} \equiv \max\{\theta \in [0,1] | G(\theta, \sqrt{\tau}) \leq p\}$.

Step 3: Unique rationalizable strategy

Let us define

$$x(\tau) \equiv \arg\min_{x\in[0,1]} G(x, \sqrt{\tau})$$

and

$$y(\tau) \equiv G(x(\tau), \sqrt{\tau}). \tag{18}$$

Following theorem of maximum, we know $y(\tau)$ is well defined and is continuous in τ. Now for all $\tau_1 < \tau_2$, we have $y(\tau_1) = G(x(\tau_1), \sqrt{\tau_1}) > G(x(\tau_1), \sqrt{\tau_2}) > G(x(\tau_2), \sqrt{\tau_2}) = y(\tau_2)$. The first inequality comes from the fact that $G(x, \sqrt{\tau})$ is decreasing in τ, and the second follows the definition of $x(\tau)$. Hence, $y(\tau)$ is decreasing with τ. Moreover, $y(0) = \lim_{\tau\to 0} y(\tau) = 1 > p$. Define

$$\tau^* \equiv \sup\{\tau | y(\tau) \geq p\}. \tag{19}$$

If $y(\infty) = \lim_{\tau\to\infty} y(\tau) < p$, then for all $\tau < \tau^*$ and all $x \in [0, 1]$, we have $G(x, \sqrt{\tau}) > p$. Otherwise, if $y(\infty) \geq p$, then $\tau^* = \infty$. Thus, if $\tau < \tau^*$, there is no $\bar{\theta}_\infty > 0$ such that $G(\bar{\theta}_\infty, \tau) = p$. Hence, $\bar{\theta}_\infty = 0$ is the only possible limit. This implies $\bar{s}_\infty = -1/(2\sqrt{\tau})$. Note that, if $s < -1/(2\sqrt{\tau})$, the unique rationalizable action is to attack the preferred regime, and when $\theta \geq 0$ agent will never receive such a signal. Also, when $\bar{s}_\infty = -1/(2\sqrt{\tau})$ is the only possible limit, it means attacking the preferred regime for any $s \geq -1/(2\sqrt{\tau})$ has been eliminated as never-best response. Therefore, the unique rationalizable strategy is not to attack the PPR. This is the unique equilibrium in which agents play monotone strategies. This implies that, when τ is sufficiently small, the preferred regime will succeed whenever $\theta \geq 0$. ■

A Dynamic Game without Future Risk

Let us assume that $\theta_2^* = 0$. For the first group of agents, if there is no future risk, then the effective fundamental strength is θ_1/α_1 and the effective private information is $s/\alpha_1 = \theta_1/\alpha_1 + \epsilon/(\alpha_1\sqrt{\tau})$. Let θ_1^*/α_1 be the equilibrium threshold. Then the threshold signal must satisfy

$$P\left(\frac{s}{\alpha_1} < \frac{s_1^*}{\alpha_1} \middle| \frac{\theta_1}{\alpha_1} = \frac{\theta_1^*}{\alpha_1}\right) = \frac{\theta_1^*}{\alpha_1}$$

Hence, $s_1^*/\alpha_1 = \theta_1^*/\alpha_1 + F^{-1}(\theta_1^*/\alpha_1)/(\alpha_1\sqrt{\tau})$. Therefore, the threshold agent believes that the preferred regime will succeed with probability $G(\theta_1^*/\alpha_1, \alpha_1\sqrt{\tau})$. If $\alpha_1 < \sqrt{\tau^*}/\sqrt{\tau} \equiv \alpha^*$ (τ^* as defined in Equation (19)), then from Lemma 2, we know that no agent will attack the PPR and there is no current risk for agents in group 1. □

Proof of Lemma 1 By definition,

$$\psi^{t-1}(h_t(x)) = h_t(x) - \sum_{u=1}^{t-1} \alpha_u F(\sqrt{\tau}(s_u^* - h_t(x))) = x.$$

Therefore, $h_t(x) - h_t(0) - (\sum_{u=1}^{t-1} \alpha_u (F(\sqrt{\tau}(s_u^* - h_t(x))) - F(\sqrt{\tau}(s_u^* - h_t(0))))) = x - 0$. Since h_t is a nondecreasing function, we have $h_t(x) \geq h_t(0)$. Thus, $F(\sqrt{\tau}(s_u^* - h_t(x))) - F(\sqrt{\tau}(s_u^* - h_t(0))) < 0$ for all $u \in \{1, 2, ..., t-1\}$. Hence, $h_t(x) - h_t(0) \leq x - 0$. □

Proof of Proposition 4 Given no future risk - i.e., $\theta_{t+1}^* = 0$, for any $\theta_t^*/\alpha_t = x$, we have shown (See Equation (12)) that

$$P\left(\frac{\theta_t}{\alpha_t} \geq x \middle| s_t^*(x), \frac{\theta_t}{\alpha_t} \geq 0\right) \geq G\left(x, \alpha_t\sqrt{\tau}\right).$$

The above inequality follows from Lemma 1.

For the same group size $\alpha_t = \alpha_1$, if $G(x, \alpha_1) > p$ for all x, then $P\left(\theta_t/\alpha_t \geq x \middle| s_t^*(x), \theta_t/\alpha_t \geq 0\right) > p$ for all x. From Proposition 3 we know that if $\alpha_1 < \alpha^*$, then $G(x, \alpha_1) > p$ for all x. Therefore, if $\alpha_t < \alpha^*$, then $P\left(\theta_t/\alpha_t \geq x \middle| s_t^*(x), \theta_t/\alpha_t \geq 0\right) > p$ for all x. From Case 1 of Proposition 2, it follows that $\theta_t^* = 0$ and $s_t^* = h_t(0) - 1/(2\sqrt{\tau})$. Thus, no agent in group t attacks the PPR if the PPR is viable when the game reaches group t. □

Proof of Theorem 1 Agents in group T face no future risk, hence $\theta_{T+1}^* = 0$. If $\alpha_t < \alpha^*$ for all $t = 1, 2 \ldots T$, then from Proposition 4 it follows that $\theta_T^* = 0$. Thus, agents in group $(T-1)$ do not face any future risk. Using Proposition 4 again we can show that $\theta_{T-1}^* = 0$. This continues until group 1. Thus, the coordination risk unravels from the end. □

Proof of Corollary 2 Recall that $\tau^* \equiv \sup\{\tau | y(\tau) \geq p\}$ and $\alpha^* = \sqrt{\tau^*}/\sqrt{\tau}$ (See Proposition 3). Hence, surely α^* is decreasing in τ. Also recall that $y(\tau)$ as defined in Equation (18) is decreasing in τ (See step 3 in Lemma 2). Therefore, τ^* and consequently α^* are decreasing in p. □

Appendix C: Application: Debt Run

Proof of Proposition 5 We will prove this Proposition in two steps. First, we prove that the conclusion of Theorem 1 holds for a more general payoff structure . Then, we show that the conclusion of Theorem 1 holds when agents are paid immediately after they attack the preferred regime and that this payoff only depends on whether the preferred regime can withstand the attack from the current group. This is precisely Proposition 5.

Step 1: We show that the conclusion of Theorem 1 holds for a general payoff structure satisfying Assumption 1 below. The payoff structure of dynamic panic-based run is a special case.

Suppose that at time $t \in \{1, 2, \ldots, T\}$ of the diffused coordination risk model, agent i's payoff u depends on her action a_{it}, the current attack from group t $\alpha_t w_t$, the future aggregate attack $w^t \equiv \sum_{u=t+1}^{T} \alpha_u w_u$ and the residual fundamental strength θ_t as follows:

$$u(a_{it} = 1, \theta_t, \alpha_t w_t, w^t) = \begin{cases} b_1(\theta_t, \alpha_t w_t, w^t) & \text{if } \theta_t \geq \alpha_t w_t + w^t \\ c_0(\theta_t, \alpha_t w_t, w^t) & \text{if } \theta_t < \alpha_t w_t + w^t \end{cases}$$

and

$$u(a_{it} = 0, \theta_t, \alpha_t w_t, w^t) = \begin{cases} c_1(\theta_t, \alpha_t w_t, w^t) & \text{if } \theta_t \geq \alpha_t w_t + w^t \\ b_0(\theta_t, \alpha_t w_t, w^t) & \text{if } \ \theta_t < \alpha_t w_t + w^t, \end{cases}$$

where $w_{T+1} = w^T = 0$. Suppose, further, that u has the properties detailed in Assumption 1.

Assumption 1 *The payoff has the following properties:*

1. *(Complementarity)* $\bar{u}(\theta_t, \alpha_t w_t, w^t) \equiv b_1(\theta_t, \alpha_t w_t, w^t) - c_1(\theta_t, \alpha_t w_t, w^t) > 0$ *and* $\underline{u}(\theta_t, \alpha_t w_t, w^t) \equiv c_0(\theta_t, \alpha_t w_t, w^t) - b_0(\theta_t, \alpha_t w_t, w^t) < 0$.

2. *(Boundedness)* $0 < \underline{n} < \bar{u}(.) < \bar{n}$ *and* $0 < \underline{m} < -\underline{u}(.) < \bar{m}$, *where* $\underline{m}$, $\bar{m}$, $\underline{n}$ *and* $\bar{n}$ *are constants.*

Proof. This step is similar to the proof in Theorem 1 except for this more general payoff structure satisfying Assumption 1. Let $\{\theta_t^*, s_t^*\}_{t=1}^T$ be the equilibrium thresholds. The definition of monotone equilibrium is similar to that in Proposition 2. Let $H(s, s_T^*)$ be the payoff difference between attacking and not attacking the preferred regime, when the agent receives private information s, public information $\theta_T \geq 0$ and expects other agents at T follow the threshold rule s_T^*. Hence,

$$H(s, s_T^*) \equiv \int_{\theta \geq h_T(\theta_T^*)} \bar{u}(\psi^{T-1}(\theta), \alpha_T w_T(\theta), w^T = 0) dF(\theta|s) + \int_{\theta < h_T(\theta_T^*)} \underline{u}(\psi^{T-1}(\theta), \alpha_T w_T(\theta), w^T = 0) dF(\theta|s). \tag{20}$$

Note that $\theta_T = \psi^{T-1}(\theta)$ and $\theta = h_T(\theta_T)$ as we have defined in the main paper. The agent who receives the threshold signal s_T^* believes that w_T is distributed as follows:

$$P(w_T \leq k | s_T^*, \theta_T \geq 0) = P\left(F\left(\sqrt{\tau}(s_T^* - \theta)\right) \leq k \middle| s_T^*, \theta \geq h_T(0) \right) = \frac{P(\theta \geq s_T^* - \frac{1}{\sqrt{\tau}} F^{-1}(k) | s_T^*)}{P(\theta \geq h_T(0) | s_T^*)} = \frac{k}{F\left(\sqrt{\tau}(s_T^* - h_T(0))\right)}.$$

Let $v(w_T, s_T^*)$ represent the fundamental strength θ such that the current aggregate attack is equal to w_T when agents at time T are following the threshold strategy s_T^*. Substituting θ by $v(w_T, s_T^*) \equiv s_T^* - F^{-1}(w_T)/\sqrt{\tau}$, we can write $H(s_T^*, s_T^*)$ (as an integral of w_T) as follows:

$$H(s_T^*, s_T^*) = \frac{1}{F\left(\sqrt{\tau}(s_T^* - h_T(0))\right)} \int_0^{\frac{\theta_T^*}{\alpha_T}} \bar{u}(\psi^{T-1}(v(w_T, s_T^*)), \alpha_T w_T, 0) dw_T + \frac{1}{F\left(\sqrt{\tau}(s_T^* - h_T(0))\right)} \int_{\frac{\theta_T^*}{\alpha_T}}^{F\left(\sqrt{\tau}(s_T^* - h_T(0))\right)} \underline{u}(\psi^{T-1}(v(w_T, s_T^*)), \alpha_T w_T, 0) dw_T$$

Next, we show that when α_T is sufficiently small, $H(s_T^*, s_T^*) > 0$ for any possible equilibrium path $(\theta_t^*)_{t=1}^{T-1}$, and (s_T^*, θ_T^*) satisfying $s_T^* = h_T^*(\theta_T) + F^{-1}(\theta_T^*/\alpha_T)/\sqrt{\tau}$. Replacing θ_T^*/α_T by $F\left(\sqrt{\tau}(s_T^* - h_T(\theta_T^*))\right)$ in $H(s_T^*, s_T^*)$, we have

$$
\begin{aligned}
H(s_T^*, s_T^*) &= \frac{1}{F\left(\sqrt{\tau}(s_T^* - h_T(0))\right)} \int_0^{F\left(\sqrt{\tau}(s_T^* - h_T(\theta_T^*))\right)} \bar{u}(\psi^{T-1}(v(w_T, s_T^*)), \alpha_T w_T, 0) dw_T \\
&+ \frac{1}{F\left(\sqrt{\tau}(s_T^* - h_T(0))\right)} \int_{F\left(\sqrt{\tau}(s_T^* - h_T(\theta_T^*))\right)}^{F\left(\sqrt{\tau}(s_T^* - h_T(0))\right)} \underline{u}(\psi^{T-1}(v(w_T, s_T^*)), \alpha_T w_T, 0) dw_T \\
&> \frac{1}{F\left(\sqrt{\tau}(s_T^* - h_T(0))\right)} \left(F\left(\sqrt{\tau}(s_T^* - h_T(\theta_T^*))\right) \times \underline{n}\right) \\
&+ \frac{1}{F\left(\sqrt{\tau}(s_T^* - h_T(0))\right)} \left(\left[F\left(\sqrt{\tau}(s_T^* - h_T(0))\right) - F\left(\sqrt{\tau}(s_T^* - h_T(\theta_T^*))\right)\right] \times (-\bar{m})\right) \\
&= (\underline{n} + \bar{m}) \left(\frac{F\left(\sqrt{\tau}(s_T^* - h_T(\theta_T^*))\right)}{F\left(\sqrt{\tau}(s_T^* - h_T(0))\right)} - \frac{\bar{m}}{\underline{n} + \bar{m}} \right).
\end{aligned}
$$

We want to show that $\frac{F\left(\sqrt{\tau}(s_T^* - h_T(\theta_T^*))\right)}{F\left(\sqrt{\tau}(s_T^* - h_T(0))\right)} > \frac{\bar{m}}{\underline{n}+\bar{m}}$ independent of the equilibrium path $(\theta_t^*)_{t=1}^{T-1}$ for all possible θ_T^* and s_T^*. Applying the inequality in $h_t(\theta_t^*) - h_t(0) \leq \theta_t^*$ (Lemma 1) and replacing s_T^* by $s_T^* = h_T(\theta_T^*) + F^{-1}(\theta_T^*/\alpha_T)/\sqrt{\tau}$, we have

$$
\begin{aligned}
\frac{F\left(\sqrt{\tau}(s_T^* - h_T(\theta_T^*))\right)}{F\left(\sqrt{\tau}(s_T^* - h_T(0))\right)} =& \frac{\frac{\theta_T^*}{\alpha_T}}{F(\sqrt{\tau}(h_T(\theta_T^*) - h_T(0)) + F^{-1}(\frac{\theta_T^*}{\alpha_T}))} \\
\geq& \frac{\frac{\theta_T^*}{\alpha_T}}{F(\alpha_T\sqrt{\tau}\frac{\theta_T^*}{\alpha_T} + F^{-1}(\frac{\theta_T^*}{\alpha_T}))} \\
=& G(\frac{\theta_T^*}{\alpha_T}, \alpha_T\sqrt{\tau}).
\end{aligned}
$$

Thus, it is sufficient to show that $G(\theta_T^*/\alpha_T, \alpha_T\sqrt{\tau}) > \bar{m}/(\underline{n} + \bar{m})$ for all possible θ_T^*. This follows directly from the proof of Lemma 2.

Given the above inequality, we can then say that there exists α^* such that for any $\alpha_T < \alpha^*$, the only possible equilibrium is that creditors will ignore their private information and never withdraw at T. Hence, $\theta_T^* = 0$ is the unique equilibrium, regardless of how agents in the prior periods attacked the preferred regime (the equilibrium specification $h_t(.)$). The rest of the argument is as in Theorem 1. ■

Step 2: The final step of Proposition 5

Proof. The payoff structure under debt run is slightly different than what we consider in the above steps. While in the above steps agents get paid in the end after all others make their decisions, in the debt run case, agents get paid immediately if they attack the preferred regime (or make withdrawals). Whether the agent gets b_0 or c_1 depends on $\theta_t < (\geq)\alpha_t w_t$ but not on $\theta_t < (\geq)\alpha_t w_t + w^t$. However, since $w^T = 0$, the subgame in period T is exactly the same as the one in Step 1. We know that when α_T is sufficiently small, $\theta_T^* = 0$ is the unique equilibrium fundamental threshold, which is independent of the equilibrium path $(\theta_t^*)_{t=1}^{T-1}$. Therefore, $w^{T-1} = w_T = 0$. The subgame in period $T-1$ is exactly the same as the case when agents receive payoff at the end since $w_T = 0$. Repeating this argument for $t < T-1$, we can obtain this unique equilibrium, $\theta_1^* = 0$ by restricting $\alpha_t < \alpha^*$ for all $t \in \{1, 2, ..., T\}$. ■

□

Appendix D: Extensions

Proof of Proposition 6 Consider agents in group 1. An agent who receives private signal s_1 believes that $\theta_1 | s_1 \sim N(s_1, 1/z_1)$, where $z_1 = \tau_1 = \tau$. Given the true state θ_1, the private signal $s_1 | \theta_1 \sim N(\theta_1, 1/\tau_1)$. In a monotone equilibrium, agents in group 1 attack the preferred regime if $s_1 < s_1^*$. Thus, the sufficient statistic $\zeta_1 = s_1$ and the threshold $\zeta_1^* = s_1^*$. Given the fundamental strength θ_1, the aggregate attack at $t = 1$ is $w_1 = \Phi(\sqrt{z_1}(\zeta_1^* - \theta_1))$.

The agents in group 2 receive additional private information :

$$s_2 = \Phi^{-1}(w_1) + \frac{1}{\sqrt{\tau_2}}\epsilon_2 = \Phi^{-1}\left(\Phi(\sqrt{z_1}(\zeta_1^* - \theta_1))\right) + \frac{1}{\sqrt{\tau_2}}\epsilon_2 = \sqrt{z_1}(\zeta_1^* - \theta_1) + \frac{1}{\sqrt{\tau_2}}\epsilon_2.$$

This can be thought of as $\left(\zeta_1^* - s_2/\sqrt{z_1}\right) = \theta_1 - \epsilon_2/\sqrt{\tau_2 z_1}$ - i.e., $\left(\zeta_1^* - s_2/\sqrt{z_1}\right)$ is a noisy information about θ_1 with precision $\tau_2 z_1$. Therefore, the updated belief is

$$\theta_1 | s_1, s_2 \sim N\left(\frac{z_1}{z_1 + \tau_2 z_1}\zeta_1 + \frac{\tau_2 z_1}{z_1 + \tau_2 z_1}\left(\zeta_1^* - \frac{1}{\sqrt{z_1}}s_2\right), \frac{1}{z_2}\right),$$

where $z_2 = z_1 + \tau_2 z_1 = \tau_1(1+\tau_2)$. Thus, the sufficient statistic is $\zeta_2 = \frac{z_1}{z_1+\tau_2 z_1}\zeta_1 + \frac{\tau_2 z_1}{z_1+\tau_2 z_1}\left(\zeta_1^* - s_2/\sqrt{z_1}\right)$ and $\zeta_2 | \theta_1 \sim N(\theta_1, 1/z_2)$. In the monotone equilibrium, agents in group 2 attack the preferred regime iff $\zeta_2 < \zeta_2^*$ and thus, $w_2 = \Phi(\sqrt{z_2}(\zeta_2^* - \theta_1))$.

Proceeding the same way, at any $t+1$, $\theta_1|\zeta_t \sim N(\zeta_t, 1/z_t)$ and $\zeta_t|\theta_1 \sim N(\theta_1, 1/z_t)$. After agents receive additional noisy private information s_{t+1} about $w_t = \Phi(\sqrt{z_t}(\zeta_t^* - \theta_1))$, their updated belief becomes

$$\theta_1|s_1, s_2, \ldots s_{t+1} \sim N(\frac{z_t}{z_t + \tau_{t+1}z_t}\zeta_t + \frac{\tau_{t+1}z_t}{z_t + \tau_{t+1}z_t}\left(\zeta_t^* - \frac{1}{\sqrt{z_t}}s_{t+1}\right), \frac{1}{z_{t+1}}),$$

where $z_{t+1} = z_t(1+\tau_{t+1}) = \tau_1 \prod_{u=2}^{t+1}(1+\tau_u)$. Thus, the sufficient statistic is $\zeta_{t+1} = \frac{z_t}{z_t+\tau_{t+1}z_t}\zeta_t + \frac{\tau_{t+1}z_t}{z_t+\tau_{t+1}z_t}\left(\zeta_t^* - s_{t+1}/\sqrt{z_t}\right)$ and $\zeta_{t+1}|\theta_1 \sim N(\theta_1, 1/z_{t+1})$. In the monotone equilibrium, agents in group $(t+1)$ attack the preferred regime iff $\zeta_{t+1} < \zeta_{t+1}^*$.

A monotone equilibrium can be characterized exactly as in Proposition 2, by replacing the sufficient statistic s_t^* with ζ_t^* and by replacing the precision of private information τ with $z_t = \tau_1 \prod_{u=2}^{t}(1+\tau_u)$. Because of cumulative learning, this precision of the aggregated information depends on how many pieces of information the agent receives, as well as on the precision of each piece of information.

If the principal makes a partition in T groups of equal sizes, then by analogy with the case without cumulative learning in Proposition 3, the precision of the private information for the last group becomes $\left(\sqrt{z_T}/T\right)^2$. If the precision z_T does not converge to infinity at a faster rate than T^2 - i.e., $\lim_{T\to\infty}\sqrt{z_T}/T \to 0$, then for sufficiently large T, $\sqrt{z_T}/T < \sqrt{\tau^*}$, where τ^* is the threshold precision below which agents ignore their private information and follow the principal's recommendation (See the proof of Corollary 2). Since, the precision of private information is lower for agents in earlier groups, the same unraveling argument applies inductively. Thus, sufficient diffusion eliminates all the coordination risk. □

Proof of Proposition 7 After an agent receives a private signal s, he updates his belief that $\theta|s \sim N(\frac{\tau_0}{\tau_0+\tau_1}\theta_0 + \frac{\tau_1}{\tau_0+\tau_1}s, \frac{1}{\tau_0+\tau_1})$. The monotone equilibrium (θ_t^*, s_t^*) can be characterized as in Proposition 2, where the recursive relation in Equation (6) becomes

$$\frac{\Phi\left(\frac{\sqrt{\tau_1}}{\sqrt{\tau_0+\tau_1}}\Phi^{-1}\left(\frac{\theta_t^*-\theta_{t+1}^*}{\alpha_t}\right) + \frac{\tau_0}{\sqrt{\tau_0+\tau_1}}(\theta_0 - h_t(\theta_t^*))\right)}{\Phi\left(\frac{\sqrt{\tau_1}}{\sqrt{\tau_0+\tau_1}}\Phi^{-1}\left(\frac{\theta_t^*-\theta_{t+1}^*}{\alpha_t}\right) + \frac{\tau_0}{\sqrt{\tau_0+\tau_1}}(\theta_0 - h_t(\theta_t^*)) + \sqrt{\tau_0+\tau_1}(h_t(\theta_t^*) - h_t(0))\right)} = p. \tag{21}$$

Comparing the indifference equation for the threshold agent with an informative prior (Equation (21)) to that with an uninformative prior (Equation (6)), we can identify the **prior effect** as $\frac{\tau_0}{\sqrt{\tau_0+\tau_1}}(\theta_0 - h_t(\theta_t^*))$.

Consider the last group. Note that, in equilibrium, $h_T(\theta_T^*) = \theta_1^*$. Hence, $\theta_0 - h_T(\theta_T^*)$ measures the difference of the expected prior strength compared to the equilibrium threshold for the preferred regime to succeed and $\tau_0/\sqrt{\tau_0+\tau_1}$ can be taken as the relative weight assigned to the prior by the threshold agent. Based on the log-concavity of Φ, it is easy to show that, everything else equal, the threshold agent will be more optimistic about the success of the preferred regime if the prior effect, $\frac{\tau_0}{\sqrt{\tau_0+\tau_1}}(\theta_0 - h_T(\theta_T^*))$, is larger. Since $h_t(\theta_t^*) = \theta_1^* \leq 1$, the worst possible prior effect at any t can be $\frac{\tau_0}{\sqrt{\tau_0+\tau_1}}(\theta_0 - 1)$.

Define $L(x, \alpha_T)$ as follows ($x \equiv \theta_T^*/\alpha_T$),

$$L(x, \alpha_T) \equiv \frac{\Phi\left(\frac{\sqrt{\tau_1}}{\sqrt{\tau_0+\tau_1}}\Phi^{-1}(x) + \frac{\tau_0}{\sqrt{\tau_0+\tau_1}}(\theta_0 - 1)\right)}{\Phi\left(\frac{\sqrt{\tau_1}}{\sqrt{\tau_0+\tau_1}}\Phi^{-1}(x) + \frac{\tau_0}{\sqrt{\tau_0+\tau_1}}(\theta_0 - 1) + \alpha_T\sqrt{\tau_0+\tau_1}x\right)}.$$

Based on the log-concavity of Φ and the properties of any possible equilibrium h_t function, we can easily prove that LHS of Equation (21) $\geq L(x, \alpha_T)$. Similar to $G(x, \alpha_T\sqrt{\tau})$, $L(x, \alpha_T)$ also has the following properties:

For any θ_0, τ_0, and τ, 1. $\lim_{x\to 0} L(x, \alpha_T) = L(x = 1, \alpha_T) = 1$; 2. $\lim_{\alpha_T \to 0} L(x, \alpha_T) = 1$ and 3. L is decreasing in α_T. Thus, as argued in Lemma 2, for any positive $p \in (0, 1)$, there exists α^* such that for any $\alpha_T < \alpha^*$, $L(x, \alpha_T) > p$ for all $x \in [0, 1]$. Hence, $\theta_T^* = 0$ when $\alpha_T < \alpha^*$. Given $\theta_T^* = 0$, we can show that $\theta_{T-1}^* = \theta_{T-2}^* = \ldots = \theta_1^* = 0$ by repeating the same argument inductively.

□

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Online Appendix

Omitted Proofs in the Appendix:

Proof of Proposition 1 For completeness, we reproduce the proof of the result from Morris and Shin (2003). Readers familiar with the global game literature should skip this proof. In this proof, we consider a more general payoff structure: our usual payoff specification is a special case of this one.

Suppose that player i's payoff function is as follows:

$$u(a_i = 1, \theta, w) = \begin{cases} b_1(\theta, w) & \text{if } \theta \geq w \\ c_0(\theta, w) & \text{if } \theta < w \end{cases}, \qquad u(a_i = 0, \theta, w) = \begin{cases} c_1(\theta, w) & \text{if } \theta \geq w \\ b_0(\theta, w) & \text{if } \theta < w. \end{cases}$$

Assumption 2 *The above payoff function has the following properties:*

1. $b_1(\theta, w) - c_1(\theta, w) = \bar{u}(\theta, w) > 0$ *and* $c_0(\theta, w) - b_0(\theta, w) = \underline{u}(\theta, w) < 0$.

2. *(Monotonicity)* $\bar{u}(\theta, w)$ *and* $\underline{u}(\theta, w)$ *are nondecreasing in* θ *and nonincreasing in* w.

3. *(Boundedness) There are constants* $\underline{m}, \bar{m}, \underline{n}, \bar{n}$ *such that* $0 < \underline{n} < \bar{u}(.) < \bar{n}$ *and* $0 < \underline{m} < -\underline{u}(.) < \bar{m}$.

Assumption 2.1 implies that when regime 1 succeeds ($\theta \geq w$), an agent will be better off if he had supported it ($\bar{u} > 0$) and vice versa. Assumption 2.2 implies that the incentive to coordinate is nonincreasing in w, while the incentive to attack is nondecreasing in w. Given the private information s, the player will choose not to attack the preferred regime if she believes the probability of success is sufficiently high, - i.e.,

$$P(\theta \geq w|s) \geq \frac{b_0(\theta,w) - c_0(\theta,w)}{b_1(\theta,w) - c_1(\theta,w) + b_0(\theta,w) - c_0(\theta,w)} = \frac{1}{1 + \frac{\bar{u}(\theta,w)}{-\underline{u}(\theta,w)}}.$$

Define $\bar{p} = \max(\frac{-\underline{u}}{\bar{u}-\underline{u}}) = \frac{\bar{m}}{\underline{n}+\bar{m}}$ and $\underline{p} = \min(\frac{-\underline{u}}{\bar{u}-\underline{u}}) = \frac{\underline{m}}{\bar{n}+\underline{m}}$. Thus, an agent will never attack regime 1 if he believes that regime 1 will succeed with a probability greater than $\bar{p}$ and will surely attack regime 1 if he believes that regime 1 will fail with a probability greater than $1 - \underline{p}$.

If an agent gets a private signal $s \geq 1 + F^{-1}(\bar{p})/\sqrt{\tau}$, he believes that the $\theta > 1$ with probability greater than $\bar{p}$. Thus, even if others always attack the preferred regime, he will not attack the preferred regime. Similarly, if an agent gets a private signal $s < -F^{-1}(1-\underline{p})/\sqrt{\tau}$, he believes that the preferred regime will fail with probability greater than $1 - \underline{p}$. Thus, even if others never attack the preferred regime, he will attack the preferred regime. Similar to Lemma 2, by iterative elimination of the never-best responses (from both sides), we get $\bar{s}_\infty$ and $\underline{s}_\infty$. The only rationalizable action for any agent is never to favor the preferred regime when $s < \underline{s}_\infty$ and never to attack when $s \geq \bar{s}_\infty$.

Thus, the worst possible equilibrium is that agents choose not to attack the preferred regime if and only if $s \geq \bar{s}_\infty$ and the best equilibrium is that agents choose not to attack the preferred regime if and only if $s \geq \underline{s}_\infty$. If $\underline{s}_\infty < \bar{s}_\infty$, there are at least two monotone equilibria. We now prove that there is actually one unique monotone equilibrium, which is the unique equilibrium for the coordination game.

Consider any possible monotone equilibrium (θ^*, s^*). For any agent $i \in [0,1]$, given that other agents attack the preferred regime if and only if $s_j < s^*(j \neq i)$, an equilibrium arises if he will take the same strategy. Given the monotone strategy s^*, the total attack at fundamental strength θ is $w(\theta, s^*) = F(\sqrt{\tau}(s^* - \theta))$. The preferred regime succeeds if and only if $\theta \geq w(\theta, s^*)$ or equivalently,

$\theta \geq \theta^*$. Define the function $H(s_i, s^*)$ to be the expected payoff difference of not attacking rather than attacking the preferred regime for agent i with private noisy signal s_i as follows:

$$H(s_i, s^*) \equiv \int_{\theta \geq \theta^*} \bar{u}(\theta, w(\theta, s^*))dF(\theta|s_i) + \int_{\theta < \theta^*} \underline{u}(\theta, w(\theta, s^*))dF(\theta|s_i). \tag{22}$$

Agent i will take the threshold strategy s^* if $H(s^*, s^*) = 0$. We want to show that there exists a unique solution s^* to the equation $H(s^*, s^*) = 0$. From $w(\theta, s^*) = F(\sqrt{\tau}(s^* - \theta))$, we can express $\theta = v(w, s^*) = s^* - F^{-1}(w)/\sqrt{\tau}$. Now w is uniformly distributed for the threshold player as

$$P(w \leq W|s^*) = P\left(F\left(\sqrt{\tau}(s^* - \theta)\right)\right) \leq W|s^*) = P(\theta \geq s^* - \frac{1}{\sqrt{\tau}}F^{-1}(W)|s^*) = W.$$

By transforming the integral from θ to w, we obtain

$$H(s^*, s^*) = \int_{\theta \geq \theta^*(s^*)} \bar{u}(\theta, w(\theta, s^*))dF(\theta|s^*) + \int_{\theta < \theta^*(s^*)} \underline{u}(\theta, w(\theta, s^*))dF(\theta|s^*) \tag{23}$$

$$= \int_0^{w(\theta^*, s^*)} \bar{u}(v(w, s^*), w)dw + \int_{w(\theta^*, s^*)}^1 \underline{u}(v(w, s^*), w)dw. \tag{24}$$

If there are two different solutions to $H(s^*, s^*) = 0$, - i.e., $H(s_1^*, s_1^*) = H(s_2^*, s_2^*) = 0$, we may assume without loss of generality that $s_1^* > s_2^*$. For any given w, $v(w, s_1^*) > v(w, s_2^*)$, and we then have

$$\bar{u}(v(w, s_1^*), w) > \bar{u}(v(w, s_2^*), w) > 0 > \underline{u}(v(w, s_1^*), w) > \underline{u}(v(w, s_2^*), w).$$

We know that, in equilibrium, $w(\theta^*, s^*) = F(\sqrt{\tau}(s^* - \theta^*)) = \theta^*$. Moreover, $w(\theta_1^*, s_1^*) = \theta_1^* > w(\theta_2^*, s_2^*) = \theta_2^*$ since $s_1^* > s_2^*$. Thus,

$$\begin{aligned} H(s_1^*, s_1^*) - H(s_2^*, s_2^*) &= \int_0^{\theta_2^*} \left(\bar{u}(v(w, s_1^*), w) - \bar{u}(v(w, s_2^*), w)\right) dw \\ &+ \int_{\theta_1^*}^1 \left(\underline{u}(v(w, s_1^*), w) - \underline{u}(v(w, s_2^*), w)\right) dw \\ &+ \int_{\theta_2^*}^{\theta_1^*} \left(\bar{u}(v(w, s_1^*), w) - \underline{u}((v(w, s_2^*), w))\right) dw > 0, \end{aligned}$$

which contradicts our assumption that $H(s_1^*, s_1^*) = H(s_2^*, s_2^*) = 0$. Hence, there is a unique monotone equilibrium. This implies that a unique strategy survives the iterated elimination of never-best responses.

□

Proof of Step 1 and Step 2 of Lemma 2: *Step 1: Iterated elimination*
If it is publicly known that $\theta \geq 0$, then the minimum signal that an agent can receive is $-1/(2\sqrt{\tau})$. If each agent believes that all the agents will ignore their respective private information and never attack the PPR - i.e., $a_i(s) = 1$ for all $s \geq -1/(2\sqrt{\tau})$, then they should not attack the PPR either for any s. Since the PPR is viable - i.e., cannot fail if nobody attacks - we cannot rule out the strategy of never attacking the PPR as a never-best response.

Next, we consider the iterated elimination of "attacking the PPR" ($a_i = 0$) as never-best responses. We start with the assumption that agents always attack the PPR. We then inductively argue that after n round of elimination, attacking the PPR is not a rationalizable action if his private signal is sufficiently high, - i.e., $s \geq \bar{s}_n$, for some $\bar{s}_n$. This, in turn, implies that the preferred regime will surely materialize if the fundamental strength is sufficiently high, - i.e., $\theta \geq \bar{\theta}_n$.

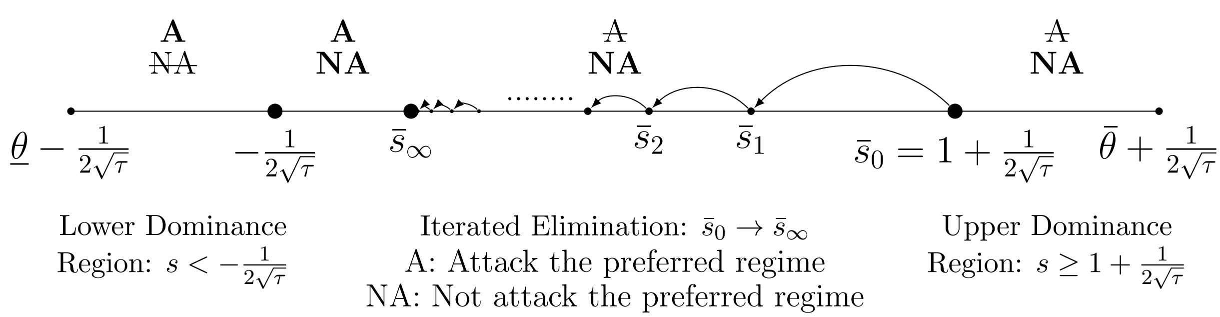

Figure 4: An Illustration: Iterated Elimination of Never Best Responses

Stage 0**:** Let us define the threshold signal $\bar{s}_0 \equiv \bar{s}$. Suppose that agents play $a_i = 1$ only when $s \geq \bar{s}_0$. Note that the preferred regime still succeeds if $\theta \geq 1$. Let us define the threshold fundamental $\bar{\theta}_0 \equiv 1$.

Stage 1**:** If an agent publicly knows $\theta \geq 0$ and receives private information s, she believes that the preferred regime succeeds with a probability of at least $P(\theta \geq \bar{\theta}_0 | s, \theta \geq 0) = F(\sqrt{\tau}(s -$

$\bar{\theta}_0))/F(\sqrt{\tau}s)$. The log-concavity of F implies $\frac{\partial}{\partial s}\left(F(\sqrt{\tau}(s-\bar{\theta}_0))/F(\sqrt{\tau}s)\right) > 0$. Recall that an agent plays $a_i = 1$ if he believes that the PPR will succeed with a probability of at least p. Let $\bar{s}_1$ be such that when $s \geq \bar{s}_1$ and the agent learns PNV, the agent believes that the PPR will succeed with a probability of at least p.

If $\bar{s}_1 > -1/(2\sqrt{\tau})$, then it must satisfy the following condition:

$$P(\theta \geq \bar{\theta}_0|\bar{s}_1, \theta \geq 0) = \frac{F(\sqrt{\tau}(\bar{s}_1 - \bar{\theta}_0)}{F(\sqrt{\tau}\bar{s}_1)} = p. \tag{25}$$

Note that when $\theta \geq 0$, the minimum signal an agent can receive is $\underline{s} = -1/(2\sqrt{\tau})$. Thus, if $F(\sqrt{\tau}(\bar{s}_1 - \bar{\theta}_0))/F(\sqrt{\tau}\bar{s}_1) > p$ for all $\bar{s}_1 \in [\underline{s}, \bar{s}]$, then $\bar{s}_1 = -1/(2\sqrt{\tau})$.

Thus, for any rational agent, $a_i = 0$ when $s \geq \bar{s}_1$ is never a best response. This is common knowledge to all agents. In the worst possible scenario, all agents decide not to attack the PPR only when $s \geq \bar{s}_1$. Hence, if the realization of fundamental strength is θ, the mass of agents who attack the PPR is $P(s < \bar{s}_1|\theta) = F(\sqrt{\tau}(\bar{s}_1 - \theta))$. Thus, the PPR fails only when

$$F(\sqrt{\tau}(\bar{s}_1 - \theta)) > \theta.$$

Note that the RHS of the above equation is increasing in fundamental strength θ, while the LHS is decreasing in fundamental strength. Let us define $\bar{\theta}_1$ as

$$P(s < \bar{s}_1|\bar{\theta}_1) = F(\sqrt{\tau}(\bar{s}_1 - \bar{\theta}_1)) = \bar{\theta}_1. \tag{26}$$

The PPR succeeds whenever $\theta \geq \bar{\theta}_1$. From Equation (26), $\bar{s}_1 = \bar{\theta}_1 + F^{-1}(\bar{\theta}_1)/\sqrt{\tau}$. By definition, $\bar{s}_0 = \bar{\theta}_0 + F^{-1}(\bar{\theta}_0)/\sqrt{\tau}$. Since $\bar{s}_1 < \bar{s}_0$, we have $\bar{\theta}_1 < \bar{\theta}_0 = 1$.

Stage 2: If $\bar{s}_1 = -1/(2\sqrt{\tau})$, then $\bar{\theta}_1 = 0$. Therefore, the iteration stops and we can say agents will never attack the PPR and the regime will succeed whenever $\theta \geq 0$. Otherwise, iterating the same process, let $\bar{s}_2$ be such that when $s \geq \bar{s}_2$ and the agents learn PNV, the agent believes that the PPR will succeed with a probability of at least p. Thus, $\bar{s}_2$ satisfies the following condition:

$$P(\theta \geq \bar{\theta}_1|\bar{s}_2, \theta \geq 0) = \frac{F(\sqrt{\tau}(\bar{s}_2 - \bar{\theta}_1)}{F(\sqrt{\tau}\bar{s}_2)} = p. \tag{27}$$

If $F(\sqrt{\tau}(\bar{s}_2 - \bar{\theta}_1)/F(\sqrt{\tau}\bar{s}_2) > p$ for all $\bar{s}_2 \in [\underline{s}, \bar{s}]$ then $\bar{s}_2 = -1/(2\sqrt{\tau})$. By comparing Equations (25) and (27), we can see that $\bar{s}_2 < \bar{s}_1$ since $\bar{\theta}_1 < \bar{\theta}_0$. Since, $F(\sqrt{\tau}(\bar{s}_2 - \bar{\theta}_1))/F(\sqrt{\tau}\bar{s}_2)$ is increasing in $\bar{s}_2$, for any rational agent, $a_i = 0$ for $s \geq \bar{s}_2$ is never a best response. This is common knowledge to all agents. In the worst possible scenario, all agents decide not to attack the PPR only when $s \geq \bar{s}_2$.

Stage $n+1$: After n rounds of elimination of never-best responses, let $\bar{s}_n$ be such that, in the worst possible scenario, an agent with public information $\theta \geq 0$ does not attack the PPR only when $s \geq \bar{s}_n$. Let $\bar{\theta}_n$ be such that, in the worst possible scenario, the PPR succeeds only when $\theta \geq \bar{\theta}_n$. Either $\bar{s}_n = -1/(2\sqrt{\tau})$ and $\bar{\theta}_n = 0$, or we recursively define $\bar{s}_{n+1}$ as, (Equation (15))

$$\frac{F(\sqrt{\tau}(\bar{s}_{n+1} - \bar{\theta}_n))}{F(\sqrt{\tau}\bar{s}_{n+1})} = p$$

or, $\bar{s}_{n+1} = -1/(2\sqrt{\tau})$ if $F(\sqrt{\tau}(\bar{s}_{n+1} - \bar{\theta}_n))/F(\sqrt{\tau}\bar{s}_{n+1}) > p$ for all $\bar{s}_{n+1} \in [\underline{s}, \bar{s}]$. Also, define $\bar{\theta}_{n+1}$ as (Equation (16))

$$\bar{\theta}_{n+1} = F(\sqrt{\tau}(\bar{s}_{n+1} - \bar{\theta}_{n+1})).$$

If $\bar{\theta}_n < \bar{\theta}_{n-1}$, through Equation (15), we have $\bar{s}_{n+1} < \bar{s}_n$. From Equation (16), we have $\bar{\theta}_{n+1} < \bar{\theta}_n$. Since $\bar{\theta}_1 < \bar{\theta}_0$, by induction, we get a decreasing sequence $\{\bar{\theta}_n\}_{n=0}^{\infty}$.

Equation (16) gives us $\bar{s}_{n+1} = \bar{\theta}_{n+1} + F^{-1}(\bar{\theta}_{n+1})/\sqrt{\tau}$. Substituting this into Equation (15), we have[18]

$$\frac{F(\sqrt{\tau}(\bar{\theta}_{n+1} - \bar{\theta}_n) + F^{-1}(\bar{\theta}_{n+1}))}{F(\sqrt{\tau}\bar{\theta}_{n+1} + F^{-1}(\bar{\theta}_{n+1}))} = p. \tag{28}$$

Let us define M: $[0,1] \to [0,1]$ such that $M(\theta)$ solves for $\bar{\theta}_{n+1}$ in Equation (28) given $\theta = \bar{\theta}_n$. If the LHS in Equation (28) is strictly larger than p for all $\bar{\theta}_{n+1} \in [0,1]$ then $M(\theta) = 0$. Since the LHS of Equation (28) is increasing in $\bar{\theta}_{n+1}$ (follows from log-concavity of F) and decreasing in $\bar{\theta}_n$, M is a well-defined and a weakly increasing function. The decreasing sequence $\{\bar{\theta}_n\}_{n=0}^{\infty}$ satisfies: 1. $\bar{\theta}_0 = 1$ and 2. $\bar{\theta}_{n+1}(\bar{\theta}_n) = M(\bar{\theta}_n)$. Note that if $\bar{\theta}_n = 0$, then for any $\bar{\theta}_{n+1} \in [0,1]$, the LHS of

[18] Note that $\theta \leq \bar{\theta}$ can be taken as another piece of public information. Taking this piece of public information into consideration, Equation (15) can be written as $P(\theta > \bar{\theta}_n | \bar{s}_{n+1}, 0 \leq \theta \leq \bar{\theta}) = \frac{F(\sqrt{\tau}(\bar{s}_{n+1} - \bar{\theta}_n)) - F(\sqrt{\tau}(\bar{s}_{n+1} - \bar{\theta}))}{F(\sqrt{\tau}(\bar{s}_{n+1})) - F(\sqrt{\tau}(\bar{s}_{n+1} - \bar{\theta}))} = p$. Combining this with Equation (28) yields $\frac{F(\sqrt{\tau}(\bar{\theta}_{n+1} - \bar{\theta}_n) + F^{-1}(\bar{\theta}_{n+1})) - F(\sqrt{\tau}(\bar{\theta}_{n+1}) - \bar{\theta} + F^{-1}(\bar{\theta}_{n+1}))}{F(\sqrt{\tau}(\bar{\theta}_{n+1}) + F^{-1}(\bar{\theta}_{n+1})) - F(\sqrt{\tau}(\bar{\theta}_{n+1}) - \bar{\theta}) + F^{-1}(\bar{\theta}_{n+1}))} = p$. Because we assume that the prior is uninformative, - i.e., $\bar{\theta} > 1 + 1/\sqrt{\tau}$ and $\bar{\theta}_{n+1} \in [0,1]$, we have $0 \leq F(\sqrt{\tau}(\bar{\theta}_{n+1} - \bar{\theta}) + F^{-1}(\bar{\theta}_{n+1})) \leq F(-1 + F^{-1}(1)) = F(-1 + 1/2) = 0$. Thus, the assumptions on F and $\bar{\theta} > 1 + 1/\sqrt{\tau}$ guarantee that we can ignore the public information $\theta \leq \bar{\theta}$.

Equation (28) is strictly larger than p. Hence, $\bar{\theta}_{n+1} = M(\bar{\theta}_n) = 0$.

For any τ, we have a decreasing sequence of thresholds $\{\bar{\theta}_n\}$, which is bounded below by 0. Therefore, the limit exists. Let $\bar{\theta}_\infty = \lim_{n\to\infty} \bar{\theta}_n$. Note that either $\bar{\theta}_\infty = 0$ or $\bar{\theta}_\infty > 0$. For any given $\tau > 0$, if $\bar{\theta}_\infty > 0$, then it must satisfy

$$\lim_{n\to\infty} \frac{F(\sqrt{\tau}(\bar{\theta}_{n+1} - \bar{\theta}_n) + F^{-1}(\bar{\theta}_{n+1}))}{F(\sqrt{\tau}\bar{\theta}_{n+1} + F^{-1}(\bar{\theta}_{n+1}))} = \frac{\bar{\theta}_\infty}{F(\sqrt{\tau}\bar{\theta}_\infty + F^{-1}(\bar{\theta}_\infty))} = G(\bar{\theta}_\infty, \sqrt{\tau}) = p, \tag{29}$$

where $G(.)$ is as defined in Equation (10).

Step 2: Worst monotone equilibrium

Suppose that $\bar{\theta}_\infty > 0$ and agents do not attack the PPR if and only if $s \geq \bar{s}_\infty = \bar{\theta}_\infty + F^{-1}(\bar{\theta}_\infty)/\sqrt{\tau}$. Then the PPR will succeed if and only if $\theta \geq \bar{\theta}_\infty$. Since $G(\bar{\theta}_\infty, \sqrt{\tau}) = p$, the agent who receives signal $\bar{s}_\infty$ is indifferent to both attacking and not attacking the PPR. Hence, $(\bar{\theta}_\infty, \bar{s}_\infty)$ constitutes a monotone equilibrium, when $\bar{\theta}_\infty > 0$. Also, $(0, -1/(2\sqrt{\tau}))$ is always a monotone equilibrium (See Corollary 3). Therefore, $(\bar{\theta}_\infty, \bar{s}_\infty)$ is a monotone equilibrium.

It is easy to see that $G(.)$ has the following properties: (1) $x \leq G(x, \sqrt{\tau}) \leq 1$, and $\lim_{x\to 1} G(x, \sqrt{\tau}) = 1$; (2) $G(x, \sqrt{\tau})$ is continuously differentiable with respect to x; (3) $G(x, \sqrt{\tau})$ is decreasing in τ. Next, we show that $\bar{\theta}_\infty = \max\{\theta \in [0,1] | G(\theta, \sqrt{\tau}) \leq p\}$. Recall from Equation (14) that $\theta^{*h} = \max\{\theta \in [0,1] | G(\theta, \sqrt{\tau}) \leq p\}$. If $\theta^{*h} > 0$, then $G(\theta^{*h}, \sqrt{\tau}) = p$ because of the continuity of G. $\theta^{*h} \in [0,1]$ is obviously a fixed point of the function M. Now, since $\theta_0 = 1 \geq \theta^{*h}$ and M is a weakly increasing fuction, $\bar{\theta}_1 = M(\bar{\theta}_0) \geq M(\theta^{*h}) = \theta^{*h}$. Similarly, $\bar{\theta}_2 = M(\bar{\theta}_1) \geq M(\theta^{*h}) = \theta^{*h}$ and thus $\bar{\theta}_n \geq \theta^{*h}$ for all n. Hence, $\bar{\theta}_\infty \geq \theta^{*h}$. Since $G(\bar{\theta}_\infty, \sqrt{\tau}) = p$ and $\theta^{*h} = \max\{\theta \in [0,1] | G(\theta, \sqrt{\tau}) \leq p\}$, we have $\bar{\theta}_\infty \leq \theta^{*h}$. Thus $\bar{\theta}_\infty = \theta^{*h}$ when $\theta^{*h} > 0$. Otherwise, if $\max\{\theta \in [0,1] | G(\theta, \sqrt{\tau}) \leq p\} = 0$, then either $G(0, \sqrt{\tau}) = p$ or $G(\theta, \sqrt{\tau}) > p$ for all θ. In either case, $\bar{\theta}_\infty = 0$. Hence, we have $\bar{\theta}_\infty = \max\{\theta \in [0,1] | G(\theta, \sqrt{\tau}) \leq p\}$. This proves the first part of Lemma 2.

Uniform Diffusion Equilibria: numerical example

We consider the policy, $(T, (\alpha))$ of uniform diffusion where $\alpha_t = 1/T$ for all $t \in \{1, 2, \ldots, T\}$. The precision of the private information is $\tau = 1$, $p = 0.7$ and $f(x) = (2 + 4x)\mathbf{1}(-0.5 \leq x < 0) + (2 - 4x)\mathbf{1}(0 \leq x \leq 0.5)$. The following table illustrates the worst possible equilibrium (highest

θ_1^*) out of all the possible equilibria. It can be seen that conforming to the theoretical prediction in the model, under sufficient diffusion, $\theta_1^* = 0$ is the unique equilibrium.

Group Size	θ_1^*	θ_2^*	θ_3^*	θ_4^*	θ_5^*
1	0.7	0	0	0	0
1/2	0.66	0.31	0	0	0
1/3	0.39	0.15	0	0	0
1/4	0.13	0	0	0	0
1/5	0	0	0	0	0

Table 1: Worst possible equilibrium $\{\theta_t^*\}_{t=1}^T$ for uniform diffusion